Comments on anomaly versus WKB/tunneling methods for calculating Unruh radiation

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ABSTRACT

In this Letter we make a critique of, and comparison between, the anomaly method and WKB/tunneling method for obtaining radiation from non-trivial spacetime backgrounds. We focus on Rindler spacetime (the spacetime of an accelerating observer) and the associated Unruh radiation since this is the prototype of the phenomena of radiation from a spacetime, and it is the simplest model for making clear subtle points in the tunneling and anomaly methods. Our analysis leads to the following conclusions: (i) neither the consistent and covariant anomaly methods gives the correct Unruh temperature for Rindler spacetime and in some cases (e.g. de Sitter spacetime) the consistent and covariant methods disagree with one another; (ii) the tunneling method can be applied in all cases, but it has a previously unnoticed temporal contribution which must be accounted for in order to obtain the correct temperature.

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1. Introduction

Recently two new methods for calculating the Hawking temperature [1] of a Schwarzschild black hole have been put forward. The first is the quasi-classical WKB method [2–6] where one calculates the tunneling rate obtained as the exponent of the imaginary part of the classical action for particles coming from the vicinity of the horizon. Developments and refinements of this method can be found in [7–9] and references therein. See also [10] for an early paper on WKB methods applied to de Sitter spacetime to calculate Hawking–Gibbons temperature [11]. More recent work applying the WKB/tunneling formalism to de Sitter space using Hamilton–Jacobi methods can be found in [12–14]. The WKB/tunneling method has also been shown to have connections to black hole thermodynamics [15–17].

The second method uses gravitational anomalies [18–20]. In this approach one places a scalar field in a Schwarzschild background and then dimensionally reduces the field equations to 1+1 dimensions near the horizon. One then discards the modes of the scalar field inside the horizon, as well as the inward directed modes on the horizon [21–23], since these are inaccessible to an outside observer. (In the original proposal of the anomaly method [18] modes behind the horizon were also considered. The simpler method of

using only near horizon and outside horizon modes was originally proposed in [21–23].) In this way one obtains an effective chiral field theory near the horizon. Such theories are known to have gravitational anomalies [24–26]. The anomaly is cancelled and general covariance is restored if one has a flux of particles coming from the horizon with the Hawking temperature. However one does not recover *directly* the Planckian spectrum from this method. In this Letter we make a critique of, and comparison between, these two methods. We mainly focus on the Rindler spacetime and the associated Unruh radiation [27]. Unruh radiation is the simple, proto-typical example of all similar effects such as Hawking radiation and Hawking–Gibbons radiation. We examine both consistent and covariant anomaly methods for two different forms of the Rindler metric. In both cases we find that neither anomaly methods gives the correct Unruh temperature.

Next we compare the consistent and covariant anomaly methods for obtaining the Gibbons–Hawking temperature of de Sitter spacetime. In this case the consistent method yields the correct Gibbons–Hawking temperature while the covariant method does not.

Given this failure of both anomaly methods we next examine the WKB/tunneling method for Rindler spacetime. Here we find that regardless of the specific form of the metric the WKB method gives the correct temperature for Rindler spacetime. However there are subtleties involved in calculating the temperature of the radiation. Here we show that there is a previously unaccounted for temporal contribution [29–31] in the WKB/tunneling method which must be taken into account in order to obtain the correct Unruh temperature.

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2. Gravitational anomaly method

The action for a massless scalar field in some background metric $g_{\mu\nu}$ can be written as

$$S[\phi] = -\frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \\ = \frac{1}{2} \int d^4x \phi \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) \phi. \quad (1)$$

By integrating out the angular variables this can be reduced to a $(1+1)$ -dimensional action [18]

$$S[\phi] = \frac{1}{2} \sum_{mn} \int d^2x \phi_{mn} \partial_\mu (\sqrt{-g} g^{\mu\nu}) \partial_\nu \phi_{mn} \quad (2)$$

where we have expanded the scalar field ϕ as

$$\phi = \sum_{mn} \phi_{mn}(t, r) e^{imy} e^{inz}. \quad (3)$$

Eliminating the scalar field modes behind the horizon as well as the ingoing modes on the horizon (these modes lead to a singular energy-momentum flux at the horizon) we are left with a $(1+1)$ -dimensional effective chiral effective theory near the horizon which is connected to a non-chiral theory outside the horizon which has both outgoing and ingoing modes. It is well known that $(1+1)$ -dimensional chiral theories exhibit a gravitational anomaly [24–26], so the energy-momentum tensor is no longer covariantly conserved (see Eq. (6.17) in [24]):

$$\nabla_\mu T_{(H)\nu}^\mu = \frac{1}{96\pi\sqrt{-g}} \epsilon^{\alpha\mu} \partial_\mu \partial_\beta \Gamma_{\alpha\nu}^\beta \equiv \frac{1}{\sqrt{-g}} \partial_\mu N_\nu^\mu. \quad (4)$$

The subscript (H) denotes the energy-momentum tensor on the horizon and g is the determinant of the $(1+1)$ -dimensional metric. Eq. (4) is the consistent gravitational anomaly. Now under general, infinitesimal coordinate transformations the variation of the $(1+1)$ -dimensional classical action is

$$\delta S = - \int d^2x \sqrt{-g} \lambda^\nu \nabla_\mu T_{\nu}^\mu. \quad (5)$$

Here $\lambda^\nu = (\lambda^t, \lambda^r)$ is the variational parameter. Normally, requiring the vanishing of the variation of the action, $\delta S = 0$, would yield energy-momentum conservation, $\nabla_\mu T_\nu^\mu = 0$, but the anomaly in (4) spoils energy-momentum conservation. We now split the energy-momentum tensor into the anomalous part on the horizon and the normal, outside the horizon part, i.e. $T_\nu^\mu = T_{(H)\nu}^\mu \Theta_H + T_{(O)\nu}^\mu \Theta_+$. $\Theta_+ = \Theta(r - r_H - \epsilon)$ is a step function with $\Theta_+ = 1$ when $r > r_H + \epsilon$ and zero otherwise. r_H is the location of the horizon and $\epsilon \ll 1$. $\Theta_H = 1 - \Theta_+$ and steps down from 1 when $r_H \leq r < r_H + \epsilon$. The subscript (O) denotes the energy-momentum tensor off the horizon. The covariant derivative of T_ν^μ is thus given by:

$$\nabla_\mu T_\nu^\mu = \frac{1}{\sqrt{-g}} \partial_\mu (\Theta_H N_\nu^\mu) \\ + \left(T_{(O)\nu}^\mu - T_{(H)\nu}^\mu - \frac{1}{\sqrt{-g}} N_\nu^\mu \right) \delta(r - r_H - \epsilon). \quad (6)$$

Using this result and considering only time-independent metric so that the partial time derivative vanishes we find that the variation of the action (5) becomes:

$$\delta S = - \int d^2x \left[\lambda^t \{ \partial_r (\Theta_H N_t^r) + (\sqrt{-g} T_{(O)t}^r - \sqrt{-g} T_{(H)t}^r - N_t^r) \delta(r - r_H) \} \right. \\ \left. + \lambda^r \{ \partial_r (\Theta_H N_r^r) + (\sqrt{-g} T_{(O)r}^r - \sqrt{-g} T_{(H)r}^r - N_r^r) \delta(r - r_H) \} \right]. \quad (7)$$

From this point on we will not explicitly write the ϵ 's. All the works on the anomaly method drop the total derivative term— $\partial_r (\Theta_H N_t^r)$ —with the justification that it is canceled by the quantum effects of the neglected ingoing modes [21,22]. In this way we find that (7), gives the following conditions:

$$\sqrt{-g} T_{(O)t}^r = \sqrt{-g} T_{(H)t}^r + N_t^r, \\ \sqrt{-g} T_{(O)r}^r = \sqrt{-g} T_{(H)r}^r + N_r^r. \quad (8)$$

We will focus on the first condition since it is the one that deals with flux. The second condition deals with pressures and for Rindler one finds that $N_t^r = 0$ so that we get just a trivial continuity condition for the radial pressure from the second condition. On the other hand we will find that for the Rindler metric the anomaly is not zero, i.e. $N_t^r \neq 0$. Thus one needs $\sqrt{-g} T_{(O)t}^r \neq \sqrt{-g} T_{(H)t}^r$. In particular the off-horizon flux must be larger by an amount $\Phi = N_t^r$ in order to cancel the anomaly and restore general covariance. Bosons with a thermal spectrum at temperature T have a Planckian distribution, i.e. $J(E) = (e^{E/T} - 1)^{-1}$ where we have taken $k_B = 1$. The flux associated with these bosons is given by [21]:

$$\Phi = \frac{1}{2\pi} \int_0^\infty E J(E) dE = \frac{\pi}{12} T^2. \quad (9)$$

If one assumes that the fluxes in (8) come from a blackbody and so have a thermal spectrum one can use (9) to give their temperature via the association $N_t^r = \Phi$. This is a second, well-known critique of the anomaly method—one has to assume the spectrum. We are now ready to apply the above results to the Rindler metric. The standard form of the Rindler metric for an observer undergoing acceleration a is

$$ds^2 = -(1 + ar)^2 dt^2 + dr^2. \quad (10)$$

In order to calculate the anomaly we need the Christoffel symbols for the metric (10). These are given by

$$\Gamma_{tt}^r = a + a^2 r, \quad \Gamma_{tr}^t = \frac{a}{1 + ar}. \quad (11)$$

Straightforwardly using these Christoffel symbols in $N_\nu^\mu = \frac{1}{96\pi} \epsilon^{\alpha\mu} \partial_\beta \Gamma_{\alpha\nu}^\beta$ one arrives at

$$N_t^r = \frac{\epsilon^{tr}}{96\pi} \partial_r \Gamma_{tt}^r = \frac{a^2}{96\pi} \quad (12)$$

combining this with $N_t^r = \Phi = \frac{\pi}{12} T^2$ one finds a temperature $T = \frac{a}{2\sqrt{2\pi}}$ which is a factor of $\frac{1}{\sqrt{2}}$ smaller than the correct Unruh temperature of $\frac{a}{2\pi}$. The source of the trouble can be traced to the fact that the standard form of the Rindler metric (10) covers the region in front to the horizon ($r = -1/a$) twice. Thus in effect the flux is spread over a larger spatial region which leads to a smaller temperature. A similar problem occurs for the Schwarzschild metric in isotropic coordinates [28] where one finds twice the correct Hawking temperature using the anomaly method. The reason is the same: isotropic coordinates double cover the region in front of the horizon and thus the flux is spread over an effectively larger region. Isotropic radial coordinates, ρ , are related to Schwarzschild radial coordinates, r , via $r = \rho(1 + \frac{M}{2\rho})^2$, where M is the mass of the black hole. From this one can see that the region $r \geq 2M$ is covered twice as ρ ranges from 0 to ∞ (the region $r \geq 2M$ is covered once by $\frac{M}{2} \leq \rho \leq \infty$ and once by $0 \leq \rho \leq \frac{M}{2}$). The reason why the Unruh temperature is reduced by $\frac{1}{\sqrt{2}}$ while the Hawking temperature is reduced by $\frac{1}{2}$ is not clear although it maybe related to the fact that the double covering of Rindler is symmetric

(we show this immediately below) while the double covering of isotropic coordinates is not.

However even the above analysis which leads to the incorrect temperature is suspect. Taking the Christoffel symbols of (11) and using them in (4) one finds that the right-hand side is zero, i.e. the anomaly vanishes (although N_t^r does not vanish) and thus there is no need to have a flux in order to cancel the anomaly at the Rindler horizon. Thus since there is no anomaly the Unruh temperature is zero according to this method. In any case for the standard form of the Rindler metric (10) whether one naively uses $N_t^r = \Phi$ or takes into account the fact that the anomaly is zero one finds that the consistent anomaly method gives the wrong temperature—either $T = \frac{a}{2\sqrt{2\pi}}$ or $T = 0$.

One might suspect that the source of the trouble is the fact that $\det(g) = 0$ for the form of the Rindler metric given in (10). This was suggested as the source of the problem for the Schwarzschild metric in isotropic coordinates [28]. In order to obtain the correct Unruh temperature via the anomaly method one should transform to a form of the Rindler metric which covers both regions—in front and behind the horizon. Such a “good” form of the Rindler metric is obtained by applying the following coordinate transformation

$$T = \frac{\sqrt{1+2ar}}{a} \sinh(at) \quad \text{and} \quad R = \frac{\sqrt{1+2ar}}{a} \cosh(at) \quad (13)$$

for $r \geq -\frac{1}{2a}$,

and

$$T = \frac{\sqrt{|1+2ar|}}{a} \cosh(at) \quad \text{and} \quad R = \frac{\sqrt{|1+2ar|}}{a} \sinh(at) \quad (14)$$

for $r \leq -\frac{1}{2a}$

to the Minkowski metric— $ds^2 = -dT^2 + dR^2$. In (13) and (14), a is the acceleration of the noninertial observer. The Rindler metric obtained after performing these coordinate transformations is the following:

$$ds^2 = -(1+2ar)dt^2 + (1+2ar)^{-1}dr^2. \quad (15)$$

Notice that in this final form we have removed the absolute value sign from around the factor $1+2ar$. Also unlike the standard form of Rindler in (10) the sign in front of the time part changes when the horizon at $r = -1/2a$ is crossed. This metric can also be found directly from the standard Rindler metric (10) by performing the following coordinate transformation

$$(1+ar_{\text{std}}) = \sqrt{|1+2ar|}. \quad (16)$$

As r ranges from $+\infty$ to $-\infty$ we find that r_{std} runs from $+\infty$ down to $r_{\text{std}} = -1/a$ and then runs back out to $+\infty$. Using the metric given by (15), the Christoffel symbols are

$$\Gamma_{tt}^r = a(1+2ar), \quad \Gamma_{tr}^t = \frac{a}{1+2ar}, \quad \Gamma_{rr}^r = -\frac{a}{1+2ar}. \quad (17)$$

Using these Christoffel symbols in (4) one finds that the anomaly vanishes (i.e. $\nabla_\mu T_\nu^\mu = 0$) so that one gets a temperature $T = 0$. Note for the Rindler metric in the form (15) $\det(g) = 1$ so we do not have the problems and ambiguity of having $\det(g) = 0$ associated with the standard form of the Rindler metric (10). If one ignores the fact that the anomaly vanishes and naively applies the formula

$$N_t^r = \frac{\epsilon^{tr}}{96\pi} \partial_r \Gamma_{tt}^r = \frac{a^2}{48\pi} = \Phi = \frac{\pi}{12} T^2, \quad (18)$$

one gets a temperature of $T = \frac{a}{2\pi}$, which is the correct Unruh temperature. However given that the anomaly explicitly vanishes we can find no justification for this procedure.

Since the above analysis was done using the consistent anomaly, which is non-covariant, one might think that this is the source of problem. However if one uses the covariant anomaly [21,33] (which as the name implies is covariant) it is immediately apparent that in any coordinate system the anomaly method will fail for Rindler spacetime. The covariant anomaly is given by

$$\nabla_\mu T_\nu^\mu = \frac{1}{96\pi\sqrt{-g}} \epsilon_{\nu\lambda} \partial^\lambda R \quad (19)$$

where R is the Ricci scalar. This method yields zero flux and zero temperature for Rindler spacetime, since Rindler has a vanishing Ricci scalar regardless of the specific form of the metric. An additional problem with the covariant method is that it gives zero Gibbons–Hawking temperature when applied to de Sitter spacetime. The 1 + 1 de Sitter in static coordinates is

$$ds^2 = -\left(1 - \frac{r^2}{\alpha^2}\right) dt^2 + \frac{dr^2}{(1 - \frac{r^2}{\alpha^2})}. \quad (20)$$

The Ricci scalar of the 1 + 1 de Sitter metric is $R = \frac{2}{\alpha^2} = \text{const.}$ Thus the covariant anomaly in (19) vanishes and the temperature of the Gibbons–Hawking radiation is wrongly given as $T = 0$. On the other hand the consistent anomaly method does give the correct Gibbons–Hawking temperature. The Christoffel symbols for the metric (20) are

$$\Gamma_{tt}^r = \frac{r(r^2 - \alpha^2)}{\alpha^4}, \quad \Gamma_{tr}^t = \frac{r}{r^2 - \alpha^2}, \quad \Gamma_{rr}^r = -\frac{r}{r^2 - \alpha^2}. \quad (21)$$

Using these one finds that the anomaly in (4) is not zero and applying $N_t^r = \Phi$ at the horizon, $r = \alpha$, yields $T = \frac{1}{2\pi\alpha}$ which is the correct Gibbons–Hawking temperature.

3. WKB-like calculation: Temporal contribution

In the previous section we found that the consistent and covariant anomaly methods did not give the correct Unruh temperature for either form of the Rindler metric (10) or (15). (The consistent anomaly method did give the correct temperature for the Rindler metric in form (15) if one ignored the fact that the anomaly was zero and naively applied $N_t^r = \Phi$.) In this section we examine how the WKB method does in calculating the Unruh temperature of Rindler spacetime.

The Hamilton–Jacobi equations give a simple way to do the WKB-like calculations. For a scalar field of mass m in a gravitational background, $g_{\mu\nu}$, the Hamilton–Jacobi equations are

$$g^{\mu\nu}(\partial_\mu S)(\partial_\nu S) + m^2 = 0, \quad (22)$$

where $S(x_\mu)$ is the action in terms of which the scalar field is $\phi(x) \propto \exp[-\frac{i}{\hbar} S(x) + \dots]$. For stationary spacetimes one can split the action into a time and spatial part, i.e. $S(x^\mu) = Et + S_0(\vec{x})$. E is the particle energy and $x^\mu = (t, \vec{x})$. Using (22) one finds [32,34] that the spatial part of the action has the general solution $S_0 = \int p_r dr$ with p_r being the radial, canonical momentum from the Hamiltonian. If S_0 has an imaginary part this indicates that the spacetime radiates and the temperature of the radiation is obtained by equating the Boltzmann factor $\Gamma \propto \exp(-\frac{E}{T})$, with the quasi-classical decay rate given by

$$\Gamma \propto \exp\left[-\text{Im}\left(\oint p_r dr\right)\right] = \exp\left[-\text{Im}\left(\int p_r^{\text{out}} dr - \int p_r^{\text{in}} dr\right)\right]. \quad (23)$$

The closed path in (23) goes across the horizon and comes back. The temperature associated with the radiation is thus given by $T = \frac{\hbar E}{\text{Im}(\oint p_r dr)}$. In almost all of the WKB/tunneling literature $\oint p_r dr$ is incorrectly replaced by $\pm 2 \int p_r^{\text{out}, \text{in}} dr$ (the latter is not invariant

under canonical transformations). The two expressions are equivalent only if the ingoing and outgoing momenta have the same magnitude. One much used set of coordinates for which this is not the case are the Painlevé–Gulstrand coordinates. These points are discussed in detail in [32,34,35].

Using the Hamilton–Jacobi equations (22) with the alternative form of the Rindler metric (15) one finds the following solution for S_0

$$S_0 = \pm \int_{-\infty}^{\infty} \frac{\sqrt{E^2 - m^2(1 + 2ar)}}{(1 + 2ar)} dr \quad (24)$$

where (+) is outgoing and (−) ingoing modes. Since the magnitude of the outgoing and ingoing S_0 are the same, using either $\oint p_r dr$ or $\pm 2 \int p_r^{\text{out, in}} dr$ gives an equivalent result. S_0 has an imaginary contribution from the pole at $r = -1/2a$. To see this explicitly we parameterize the semi-circular contour near $r = -1/2a$ by $r = -\frac{1}{2a} + \epsilon e^{i\theta}$ where $\epsilon \ll 1$ and θ goes from 0 to π for the ingoing path and π to 2π for the outgoing path. With this parameterization the contribution to the integral in (24) coming from the pole is

$$S_0 = \pm \int \frac{\sqrt{E^2 - m^2 \epsilon e^{i\theta}}}{2a \epsilon e^{i\theta}} i \epsilon e^{i\theta} d\theta = \pm \frac{i\pi E}{2a}. \quad (25)$$

In the second expression we have taken the limit $\epsilon \rightarrow 0$. Using this result in (23) apparently gives twice the correct Unruh temperature.

At first glance the standard form of the Rindler metric (10) appears to give the correct Unruh temperature. Using the Hamilton–Jacobi equations one finds the following solution for S_0

$$S_0 = \pm \int_{-\infty}^{\infty} \frac{dr_{\text{std}}}{1 + ar_{\text{std}}} \sqrt{E^2 - m^2(1 + ar_{\text{std}})^2}. \quad (26)$$

In this case it appears as if the contour integration of (26) around the pole at $r = -1/a$ would yield value $S_0 = \pm \frac{i\pi E}{2a}$. However, since the integrals in (24) and (26) are related by the coordinate transform (16) (which is just a change of variables) the value of the integral should be the same. In detail using (16) one finds that the parameterization of the contour in (25) becomes $1 + ar_{\text{std}} = \sqrt{\epsilon} e^{i\theta/2}$. From this one sees that the semi-circular contour of (25) gets transformed into a quarter circle (i.e. one must transform both the integrand and the measure). In terms of residue this means that for (25) one has $i\pi \times \text{Residue}$ while for (26) one has $i\frac{\pi}{2} \times \text{Residue}$. Thus the imaginary contributions to S_0 are the same for both (25) and (26) namely $S_0 = \pm \frac{i\pi E}{2a}$. This subtlety in the transformation of the contour is exactly parallel to what occurs for the Schwarzschild metric in the Schwarzschild form versus the isotropic form [32,34]. Thus we have an apparent factor of two discrepancy for calculating the Unruh temperature using the WKB/tunneling method. A possible resolution of this factor of two was given in [36] where an integration constant was inserted into expressions like (26) or (24) and then adjusted so as to obtain the desired answer. This resolution lacked any physical motivation for choosing the specific value of the imaginary part of the integration constant.

The actual resolution to this discrepancy is that there is a contribution coming from the Et part of $S(x_\mu)$ in addition to the contribution coming from S_0 [29–31]. The source of this temporal contribution can be seen by noting that upon crossing the horizon at $r = -1/2a$, the t, r coordinates reverse their time-like/space-like character. In more detail when the horizon is crossed one can see from Eqs. (13) and (14) that the time coordinate changes as $t \rightarrow t - \frac{i\pi}{2a}$ (along with a factor of i coming from the square root). Thus when the horizon is crossed there will be an imaginary contribution coming from the Et term of $S(x_\mu)$ of the form

$\text{Im}(E\Delta t) = -\frac{\pi E}{2a}$. For a round trip one will have a contribution of $\text{Im}(E\Delta t)_{\text{round-trip}} = -\frac{\pi E}{a}$. Adding this temporal contribution to the spatial contribution from (23) now gives the correct Unruh temperature for all forms of the Rindler metric using the WKB/tunneling method.

As a final note in addition to obtaining the Unruh temperature via (23) it is also possible to use the detailed balance method of [2] to obtain the correct Unruh temperature [37]. For detailed balance one sets $P_{\text{emission}}/P_{\text{absorption}} = \exp(-\frac{E}{T})$ where $P_{\text{emission, absorption}} = |\phi_{\text{out, in}}|^2 = \exp[-2\text{Im} \int p_r^{\text{out, in}} dr]$. One should add the temporal part to this but since the temporal part is the same for outgoing and ingoing paths (emission and absorption) and since the formula involves the ratio $P_{\text{emission}}/P_{\text{absorption}}$ the temporal part will cancel out. This explains why the detailed balance method was able to apparently give the correct result while ignoring the temporal part. However, as point out in [37] one should have the physical condition, $P_{\text{absorption}} = 1$, since classically there is no barrier for an ingoing particle to cross the horizon. The condition is only achieved when one takes into account the temporal contribution.

4. Conclusion

In this Letter we have made a comparison and critique of the anomaly and WKB/tunneling methods of obtaining radiation from a given spacetime. For Rindler spacetime we found that both the consistent and covariant anomaly method gave an incorrect Unruh temperature of $T = 0$ since in both cases the anomaly vanished. In the case of the consistent anomaly method if one ignored the vanishing of the anomaly and naively applied $N_t^r = \Phi$ one obtained an incorrect Unruh temperature of $T = \frac{a}{2\sqrt{2}\pi}$ for the form of the Rindler metric in (10) and the correct Unruh temperature of $T = \frac{a}{2\pi}$ for the form of the Rindler metric in (15). However we cannot find a justification for this naive application of $N_t^r = \Phi$ in the case of (15) since by (4) $\nabla_\mu T_{(H)\nu}^\mu = 0$. We also examined a problem with the covariant anomaly method in connection with Gibbons–Hawking radiation of de Sitter spacetime. Since de Sitter spacetime has a constant Ricci scalar the covariant anomaly (19) is zero. Thus the covariant anomaly gives a Gibbons–Hawking temperature of zero. On the other hand the consistent anomaly is non-zero and gives the correct Gibbons–Hawking temperature.

The WKB/tunneling method works for any form of the metric for Rindler spacetime, but there are subtle features. In particular there is a temporal contribution to $S(x_\mu)$ coming from a change in the time coordinate upon crossing the horizon. In addition there is the question of whether one should exponentiate $\oint p_r dr$ or $\pm 2 \int p_r^{\text{out, in}} dr$ to get the correct decay rate. This confusion has led to a wrong factor of two in calculating, for example, the Hawking temperature [32,34]. There was an ad hoc attempt at resolving this factor of two by inserting an integration constant [36] into expressions like (26) or (24) and then adjusting to get the expected answer. Physically this resolution lacked motivation. In this Letter we have shown that the arbitrarily adjusted integration constant essentially plays the role of the temporal contribution discussed above. Once this temporal contribution is taken into account one obtains the correct temperature regardless of which form of the metric is used. Although we have focused on Rindler spacetime and Unruh radiation, our results should be extendable to other spacetimes which exhibit Hawking-like radiation.

Recently there has been work which attempts to connect the WKB/tunneling method and the anomaly method [38]. The idea behind this unification of the two methods is that some anomalies can be viewed as the effect of spectral flow of the energy levels. This spectral flow is analogous to tunneling thus giv-

ing the connection. In the present work we have shown that the both anomaly methods fail for Rindler spacetime while the WKB/tunneling method recovers the correct Unruh temperature. Further the covariant anomaly method fails for de Sitter spacetime while the consistent anomaly method and WKB/tunneling method work. The results of this work indicate that the connection between the anomaly method and the WKB/tunneling method is not valid for all spacetimes.

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References

- [1] S.W. Hawking, *Commun. Math. Phys.* 43 (1975) 199.
- [2] K. Srinivasan, T. Padmanabhan, *Phys. Rev. D* 60 (1999) 24007; S. Shankaranarayanan, T. Padmanabhan, K. Srinivasan, *Class. Quantum Grav.* 19 (2002) 2671.
- [3] P. Kraus, F. Wilczek, *Nucl. Phys. B* 433 (1995) 403.
- [4] G.E. Volovik, *JETP Lett.* 69 (1999) 662.
- [5] V.A. Berezin, A. Boyarsky, A.Yu. Neronov, *Gravit. Cosmol.* 5 (1999) 16.
- [6] M.K. Parikh, F. Wilczek, *Phys. Rev. Lett.* 85 (2000) 5042; M.K. Parikh, *Int. J. Mod. Phys. D* 13 (2004) 2351.
- [7] A.J.M. Medved, E.C. Vagenas, *Mod. Phys. Lett. A* 20 (2005) 1723; A.J.M. Medved, E.C. Vagenas, *Mod. Phys. Lett. A* 20 (2005) 2449; M. Arzano, A.J.M. Medved, E.C. Vagenas, *JHEP* 0509 (2005) 037; E.C. Vagenas, *Nuovo Cimento B* 117 (2002) 899.
- [8] M. Angheben, M. Nadalini, L. Vanzo, S. Zerbini, *JHEP* 0505 (2005) 014; M. Nadalini, L. Vanzo, S. Zerbini, *J. Phys. A* 39 (2006) 6601.
- [9] R. Banerjee, B.R. Majhi, *JHEP* 0806 (2008) 095; R. Banerjee, B.R. Majhi, *Phys. Lett. B* 662 (2008) 62.
- [10] R. Brout, Ph. Spindel, *Nucl. Phys. B* 348 (1991) 405.
- [11] G.W. Gibbons, S.W. Hawking, *Phys. Rev. D* 15 (1977) 2738.
- [12] Y. Sekiwa, arXiv:0802.3266.
- [13] G.E. Volovik, arXiv:0803.3367 [gr-qc].
- [14] A.J.M. Medved, arXiv:0802.3796 [hep-th].
- [15] T. Pilling, *Phys. Lett. B* 660 (2008) 402.
- [16] R. Banerjee, B.R. Majhi, S. Samanta, *Phys. Rev. D* 77 (2008) 124035.
- [17] B. Zhang, Q.-y. Cai, M.-s. Zhao, *Phys. Lett. B* 665 (2008) 260.
- [18] S.P. Robinson, F. Wilczek, *Phys. Rev. Lett.* 95 (2005) 011303.
- [19] E.C. Vagenas, S. Das, *JHEP* 0610 (2006) 025; S. Das, S.P. Robinson, E.C. Vagenas, *Int. J. Mod. Phys. D* 17 (2008) 533.
- [20] G.E. Volovik, gr-qc/9510001.
- [21] S. Iso, H. Umetsu, F. Wilczek, *Phys. Rev. Lett.* 96 (2006) 151302.
- [22] K. Murata, J. Soda, *Phys. Rev. D* 74 (2006) 044018.
- [23] S. Iso, H. Umetsu, F. Wilczek, *Phys. Rev. D* 74 (2006) 044017.
- [24] R.A. Bertlmann, E. Kohlprath, *Ann. Phys.* 288 (2001) 137.
- [25] S.M. Christensen, S.A. Fulling, *Phys. Rev. D* 15 (1977) 2088.
- [26] L. Alvarez-Gaume, E. Witten, *Nucl. Phys. B* 234 (1983) 269.
- [27] W.G. Unruh, *Phys. Rev. D* 14 (1976) 870.
- [28] S.-Q. Wu, J.-J. Peng, Z.-Y. Zhao, arXiv:0803.1338 [hep-th].
- [29] E.T. Akhmedov, T. Pilling, D. Singleton, arXiv:0805.2653 [gr-qc], GRF honorable mention essay, *Int. J. Mod. Phys. D*, in press.
- [30] V. Akhmedova, T. Pilling, A. de Gill, D. Singleton, *Phys. Lett. B* 666 (2008) 269.
- [31] T.K. Nakamura, arXiv:0706.2916 [hep-th].
- [32] E.T. Akhmedov, V. Akhmedova, T. Pilling, D. Singleton, *Int. J. Mod. Phys. A* 22 (2007) 1705.
- [33] R. Banerjee, S. Kulkarni, *Phys. Rev. D* 77 (2008) 024018; R. Banerjee, S. Kulkarni, *Phys. Lett. B* 659 (2008) 827.
- [34] E.T. Akhmedov, V. Akhmedova, D. Singleton, *Phys. Lett. B* 642 (2006) 124.
- [35] B.D. Chowdhury, *Pramana* 70 (2008) 593.
- [36] P. Mitra, *Phys. Lett. B* 648 (2007) 240.
- [37] R. Banerjee, B.R. Majhi, D. Roy, arXiv:0901.0466 [hep-th].
- [38] S. Ghosh, arXiv:0804.2999v2 [hep-th].