

### 3.1: Functions

1. For  $f(x) = 2 - 3x - 5x^2$ , find  $\frac{f(x+h) - f(x)}{h}$  and simplify completely.

2. Find the domain of each of the following functions. Express your answers using interval notation.

a.  $f(x) = \frac{4}{\sqrt{x-9}}$

b.  $g(x) = \frac{x-3}{x^2-14x+24}$

c.  $h(x) = x^2 - 4x - 5$

3. If a rock falls from a height of 108 meters on the planet Jupiter, its height  $H$  (in meters) after  $t$  seconds is approximated by  $H(t) = 108 - 12t^2$ .

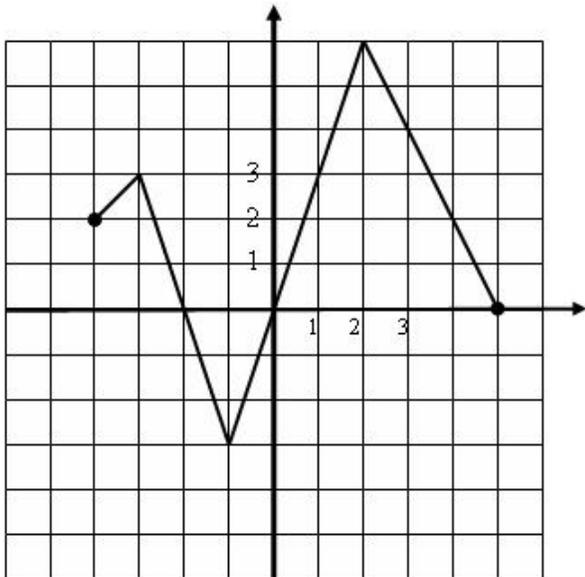
a. What is the height of the rock after 1.5 seconds?

b. At what time  $t$  will the rock strike the ground?

c. What is the implied domain of this function?

### 3.2: The Graph of a Function

1. Use the graph of the function  $f$  below to answer the following:



a. What is  $f(3)$ ?

b. What is  $f(-1)$ ?

c. For what values of  $x$  is  $f(x) = 0$ ?

d. For what values of  $x$  is  $f(x) < 0$ ?

e. For what value of  $x$  does  $f(x) = 6$ ?

f. What is the domain of  $f$ ?

g. What is the range of  $f$ ?

h. How many times does the line  $y = \frac{5}{2}$  intersect the graph of  $f$ ?

2. Let  $f(x) = \frac{x^2 + 2}{x + 4}$

a. Is the point  $(1, \frac{3}{5})$  on the graph of  $f$ ?

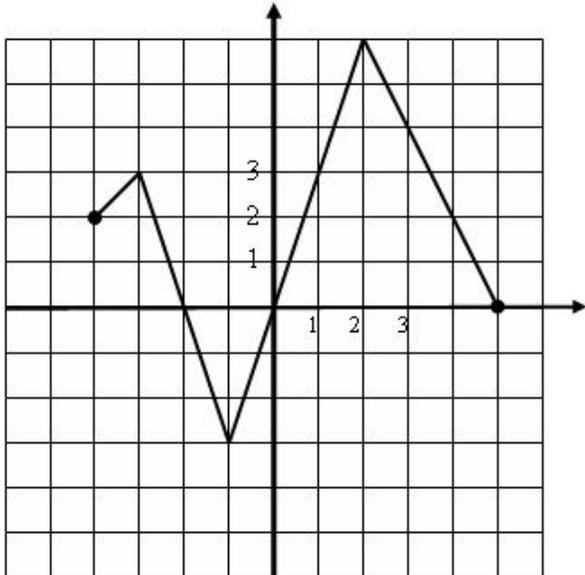
b. If  $x = 0$ , what is  $f(x)$ ? What corresponding point is on the graph of  $f$ ?

c. If  $f(x) = \frac{1}{2}$ , what corresponding points are on the graph of  $f$ ?

d. Find the  $x$  and  $y$  intercepts of the graph of  $f$ .

### 3.3: Properties of Functions

1. Use the graph of the function  $f$  below to answer the following:



a. What is the average rate of change of  $f$  on the interval  $[-4, 1]$ ?

b. For which values of  $x$  does  $f$  have a local maximum?

c. What is a local minimum value of  $f$ ?

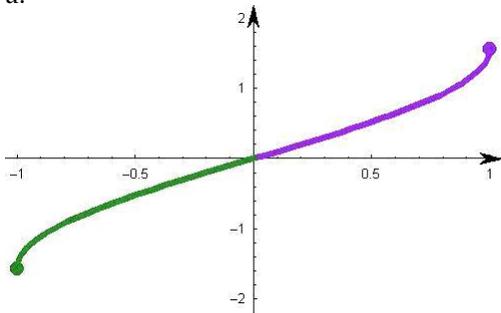
d. On what interval(s) is  $f$  increasing?

e. On what interval(s) is  $f$  decreasing?

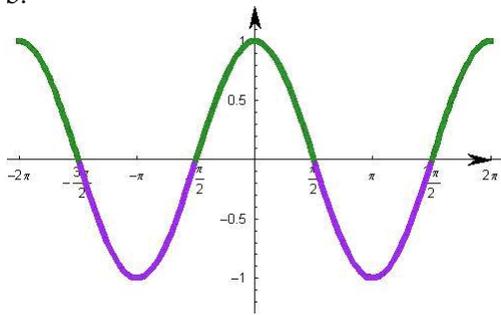
f. Is  $f$  odd, even, or neither?

2. Identify each of the following functions as odd, even, or neither:

a.



b.



c.  $f(x) = x^3 - x$

d.  $f(x) = x^4 - 9$

3. Let  $f(x) = \sqrt[3]{1-x}$ .

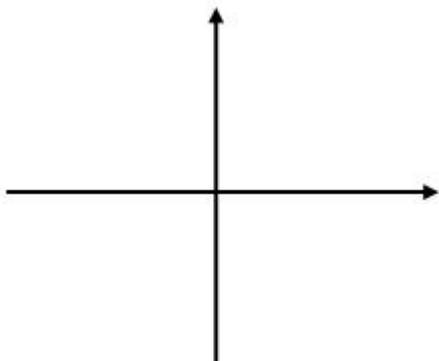
a. Find the average rate of change of  $f$  on the interval  $[-7, 9]$ .

b. Find the average rate of change of  $f$  on the interval  $[-24, 0]$ .

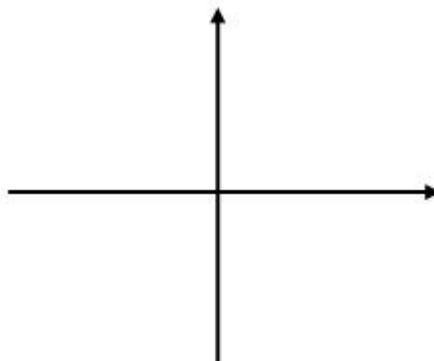
### 3.4: Library of Functions; Piecewise-Defined Functions

1. Sketch the graphs of each of the following functions. Label at least 3 points on each graph.

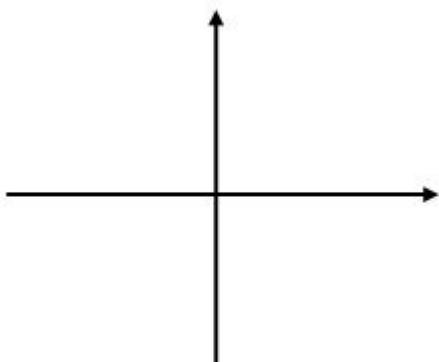
$$f(x) = x^2$$



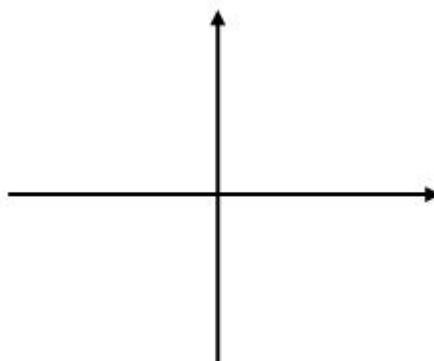
$$f(x) = \sqrt[3]{x}$$



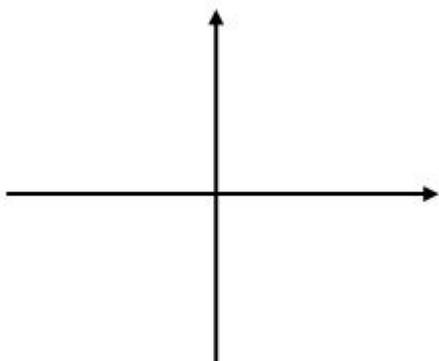
$$f(x) = x^3$$



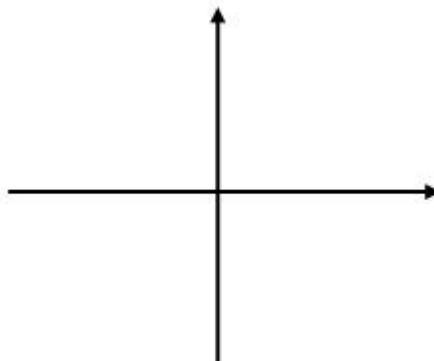
$$f(x) = \frac{1}{x}$$



$$f(x) = \sqrt{x}$$



$$f(x) = |x|$$

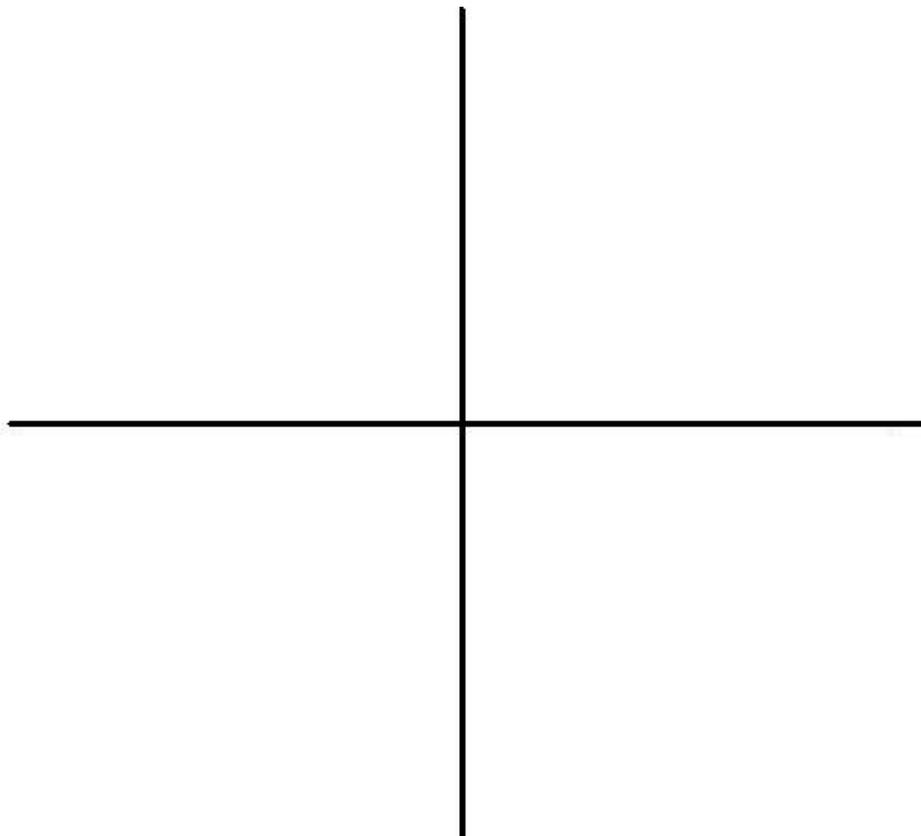


2. Let  $f(x) = \begin{cases} \frac{1}{x} & \text{if } x > 0 \\ |x| & \text{if } x \leq 0 \end{cases}$

a. Find  $f(-5)$

b. Find  $f(10)$

c. Sketch the graph of  $f$ . Label at least 3 points in the graph.

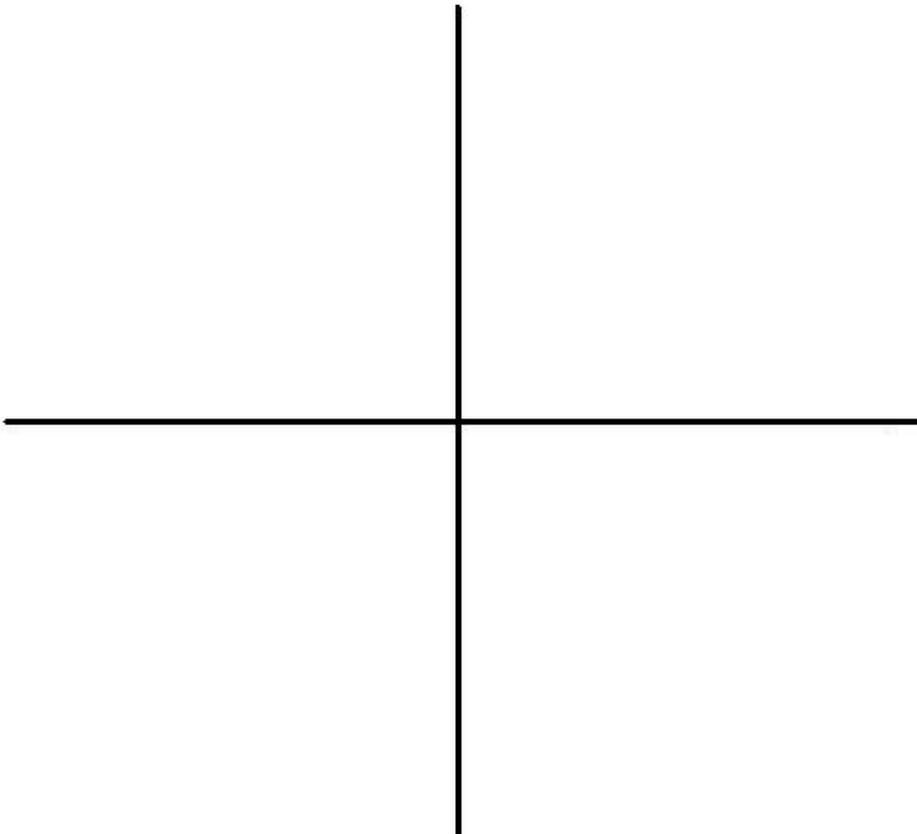


3. Let  $f(x) = \begin{cases} 5 & \text{if } x < -3 \\ 2 - x & \text{if } -3 \leq x < 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$

a. Find  $f(-2)$

b. Find  $f(-5)$

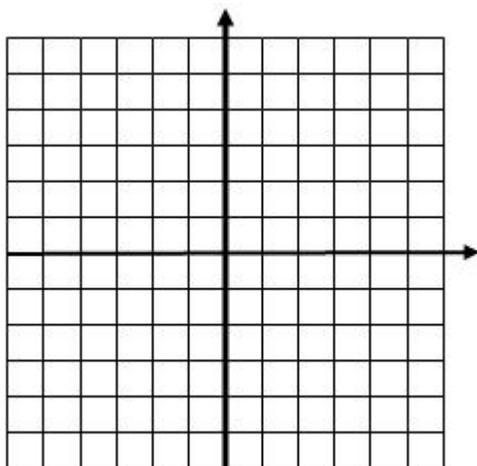
c. Sketch the graph of  $f$ . Label at least 3 points in the graph.



### 3.5: Graphing Techniques: Transformations

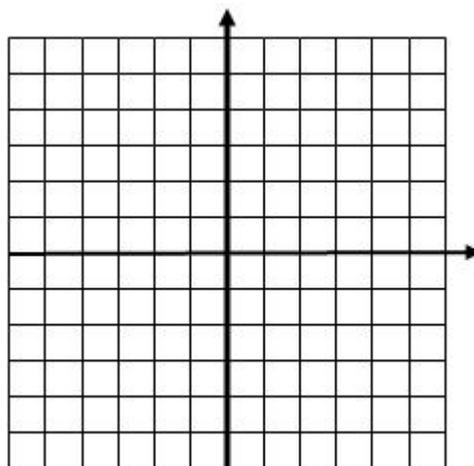
Problems 1-3: Graph each of the following functions using the techniques of shifting, compressing, stretching, and/or reflecting. Start by graphing a basic function and then show each stage of the transformation. Label at least 3 points on the final graph.

1.  $f(x) = \frac{1}{2}(x + 2)^2 - 3$



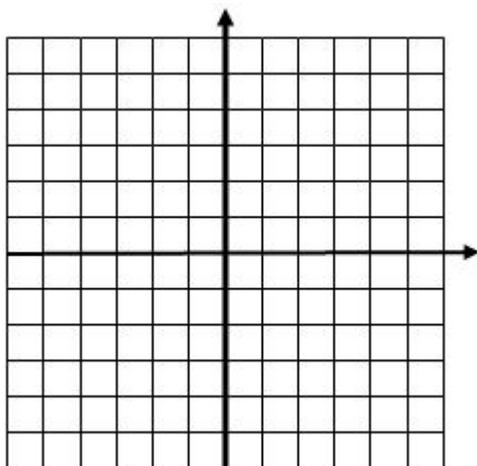
basic function:

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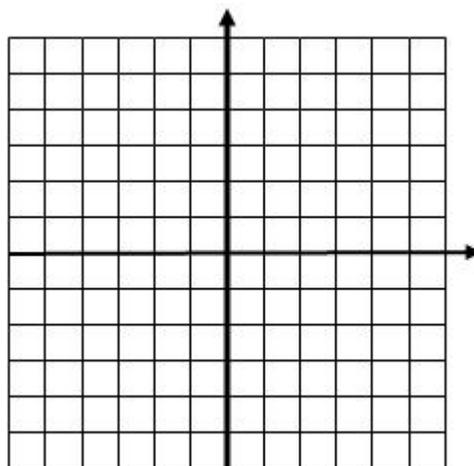
after 1st transformation:

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after 2nd transformation:

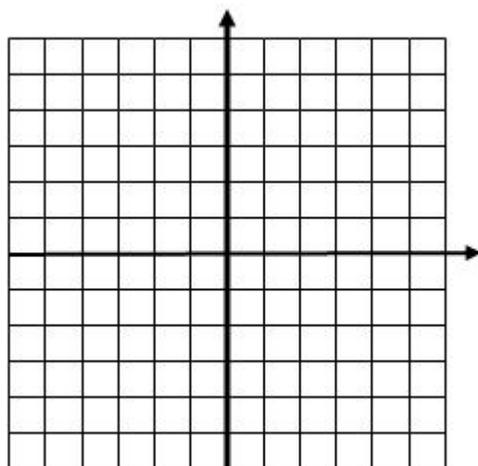
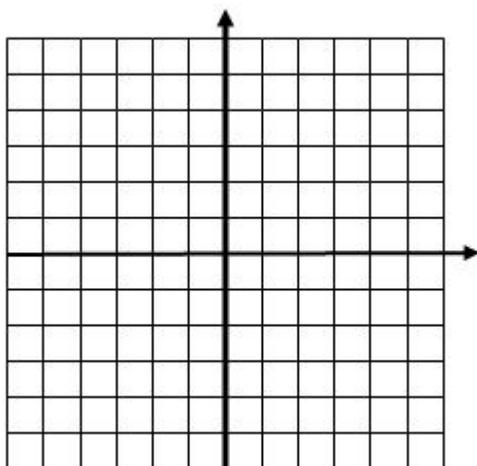
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after 3rd transformation (final answer):

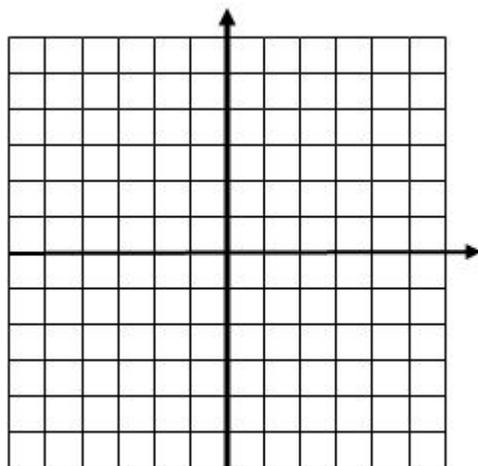
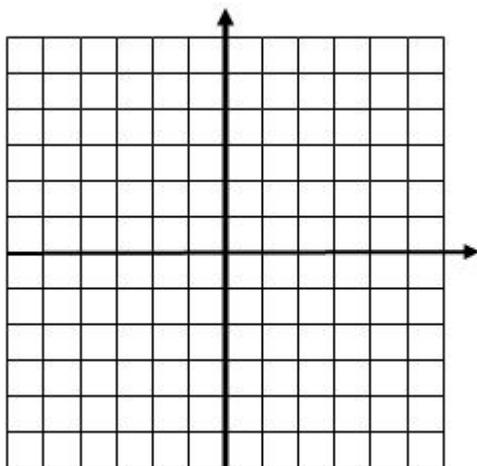
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2.  $f(x) = -\sqrt{-x} + 2$



basic function:

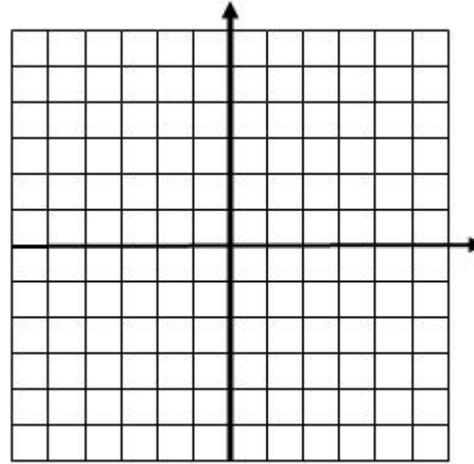
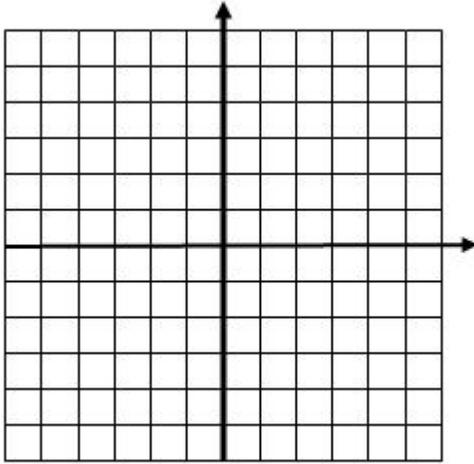
after 1st transformation:



after 2nd transformation:

after 3rd transformation (final answer):

3.  $f(x) = -\frac{1}{x-2} + 1$



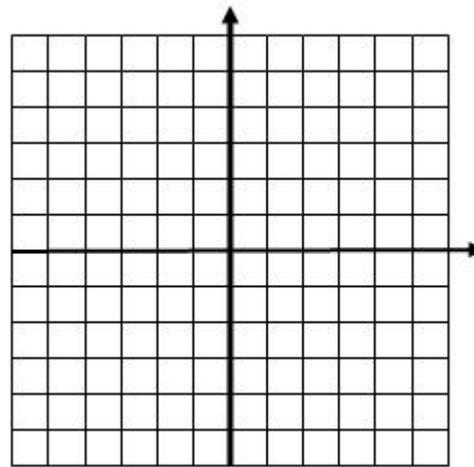
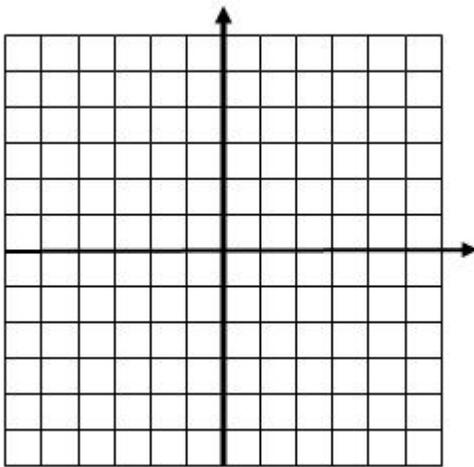
basic function:

after 1st transformation:

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after 2nd transformation:

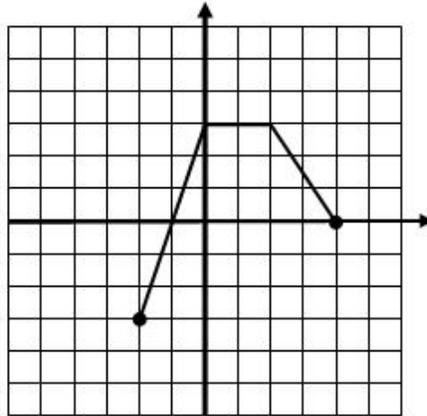
after 3rd transformation (final answer):

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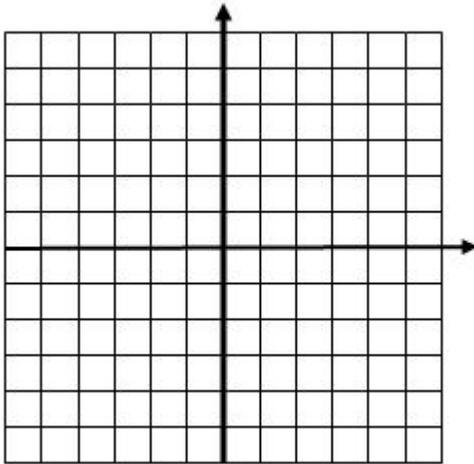


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4. The graph of a function  $F$  is given. Use this graph to graph each of the following below. Label four points on each graph.

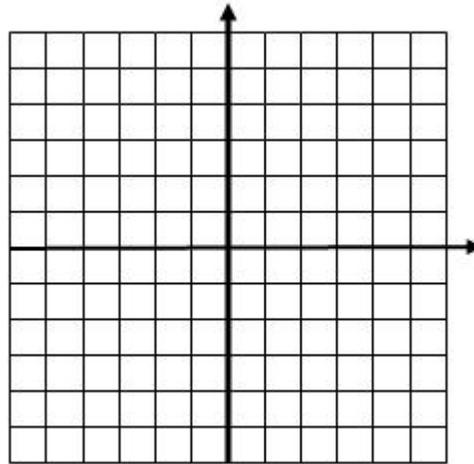


a.



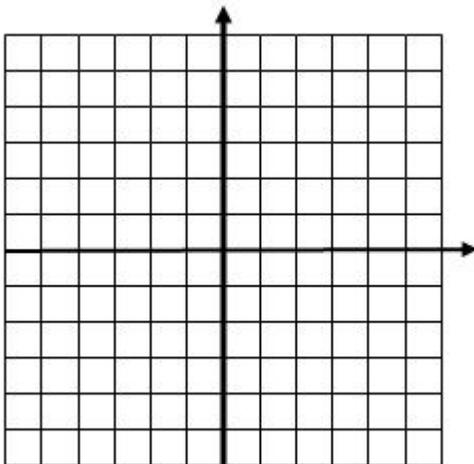
$$F(-x)$$

b.



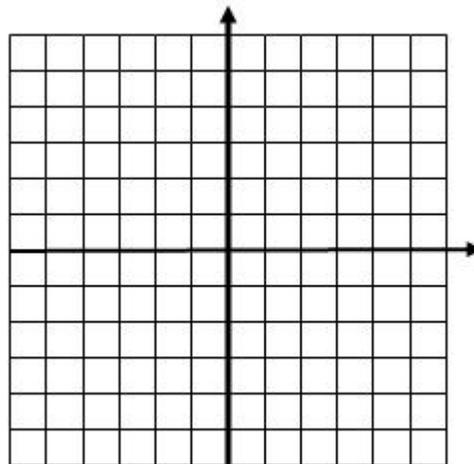
$$-F(x)$$

c.



$$\frac{1}{3}F(x)$$

d.



$$F(x+2) - 1$$

5. Write the formula for the function that is finally graphed after the following transformations are applied to the graph of  $y = |x|$  :

1st transformation: reflect about the x-axis

2nd transformation: shift right 3 units

3rd transformation: shift down 2 units

6. Write the formula for the function that is finally graphed after the following transformations are applied to the graph of  $y = \sqrt{x}$  :

1st transformation: shift up 3 units

2nd transformation: reflect about the x-axis

3rd transformation: vertically stretch by a factor of 2

### 3.6: Mathematical Models: Building Functions

1. Let  $P = (x, y)$  be a point on the graph of  $y = x^2 - 1$ .

a. Express the distance  $d$  from  $P$  to the point  $(0, 3)$  as a function of  $x$ . (Simplify your answer.)

b. What is  $d$  if  $x = 0$ ? Answer this question *without* using your function. Then use your function to verify your answer.

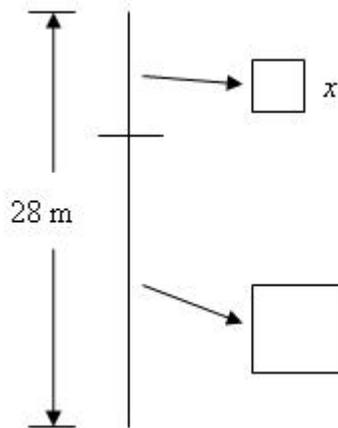
2. A rectangle with width  $x$  has a perimeter of 24 inches.

a. Express the length of the rectangle  $l$  as a function of  $x$ .

b. Find the domain of  $l$ . Express your answer in interval notation.

c. Express the area of the rectangle  $A$  as a function of  $x$ .

3. A wire 28 meters long is to be cut into two pieces. The two pieces will then be shaped into squares as shown below. Express the total area of the two squares as a function of  $x$ , where  $x$  is the length of the side of one of the squares.



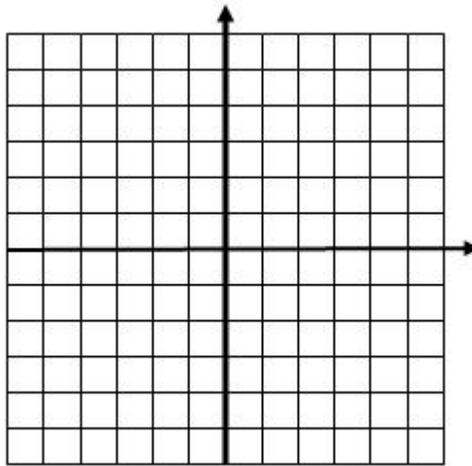
4. Two cars leave an intersection at the same time. One is headed due north at a constant speed of 20 miles per hour and the other is headed due east at a constant speed of 30 miles per hour. Express the distance  $d$  (in miles) between the cars as a function of time  $t$  (in hours). Let  $t = 0$  be the moment the cars leave the intersection.

## 4.1: Linear Functions and Their Properties

1. Let  $f(x) = \frac{3}{2}x - 1$

a. Determine the  $y$ -intercept and the slope of the graph of  $f$ .

b. Use the slope and  $y$ -intercept to sketch the graph of  $f$ . Label 3 points on the graph.



c. Find the average rate of change of  $f$  on the interval  $[2,3]$ .

d. Is  $f$  increasing, decreasing, or constant?

2. Determine for each of the following functions:

i. whether the function is increasing, decreasing, or constant

ii. the average rate of change of the function on the interval  $[\frac{1}{6}, \frac{1}{5}]$

(answer both questions **without** sketching the graph and **without** any calculations)

a.  $f(x) = -\frac{3}{5}x + 2$

i.

ii.

b.  $g(x) = 5$

i.

ii.

c.  $h(x) = \frac{1}{3}x + 4$

i.

ii.

3. Some scientists believe that the average surface temperature of the earth has been rising steadily. Suppose the average surface temperature is given by

$$T(t) = 0.02t + 8.50$$

where  $T$  is temperature in degrees Celsius and  $t$  is years since 1900.

a. What does the slope of this function represent?

b. What does the T-intercept of this function represent?

c. Use the function to predict the average global surface temperature in 2100.

4. Suppose that a small company has just purchased a new computer for \$4000. The company estimates that in four years the computer will be worth \$200. Assume the value of the computer is depreciating linearly.

a. Write a function that expresses the value  $V$  of the computer as a function of time  $t$ , where  $t$  is the time in *years* since the computer was purchased.

b. According to this model, what is the value of the computer six *months* from the time it was purchased?

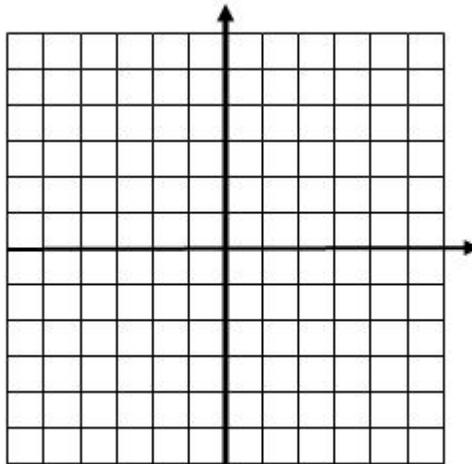
c. According to this model, when will the computer be worthless?

### 4.3: Quadratic Functions and Their Properties

1. Let  $f(x) = -2x^2 + 8x - 3$

a. Use completing the square to express  $f$  in the form  $f(x) = a(x - h)^2 + k$ .

b. Sketch the graph of  $f$ . Label the vertex,  $y$ -intercept, and one additional point.



c. Determine the domain and range of  $f$ .

d. Determine the intervals on which  $f$  is increasing and decreasing.

2. Let  $f(x) = -2x^2 + 2x + 4$

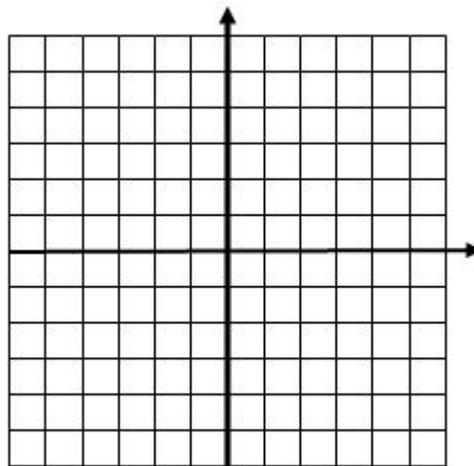
a. Use the formula  $x = -\frac{b}{2a}$  to find the vertex of the graph of  $f$ .

b. Does  $f$  have a maximum or minimum value? Explain how you know.

c. What is the maximum/minimum value of  $f$ ?

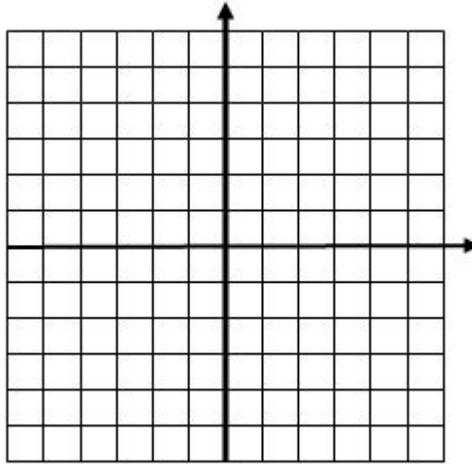
d. What is the equation of the axis of symmetry of the graph of  $f$ ?

e. Sketch the graph of  $f$ . Label the vertex,  $y$ -intercept, and  $x$ -intercepts.



3. The graph of a quadratic function contains the points  $(1, 0)$ ,  $(3, -4)$ , and  $(6, 5)$ . Its axis of symmetry has the equation  $x = 3$ .

a. Use symmetry to sketch the graph of  $f$ . Label 5 points.



b. Find the formula for  $f$ .

4. Use completing the square to express each of the following in the form  $f(x) = a(x - h)^2 + k$ .

a.  $f(x) = x^2 - 8x + 1$

b.  $f(x) = 3x^2 + 6x + 7$

## 4.4: Quadratic Models; Building Quadratic Models from Data

1. A ball is thrown directly upward. Its height (in feet) after  $t$  seconds is given by  $y = 40t - 16t^2$ .

a. What is the height of the ball after 1 second?

b. How long does it take the ball to hit the ground?

c. What is the maximum height attained by the ball?

d. What is the practical domain of this function?

2. An olympic springboard contestant dives into a swimming pool. Her path is given by

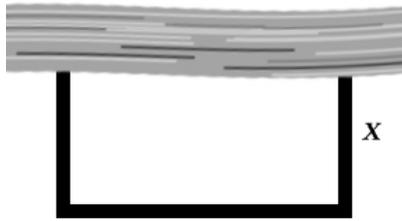
$y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$  where  $y$  is her height above the water (in feet) and  $x$  is her horizontal distance from the end of the diving board (in feet).

a. What is the maximum height reached by the diver?

b. How far from the end of the diving board is the diver when the maximum height is reached?

c. What is the height of the diving board above the water?

3. A rectangular field is to be enclosed with 280 yards of fencing. One side of the field faces a river and does not need fencing.



a. Express the area of the field  $A$  as a function of its width  $x$ .

b. For what value of  $x$  will the area be the greatest?

c. What is the greatest area that can be enclosed?

## 5.1: Polynomial Functions and Models

1. Let  $P(x) = -x^2(x - 3)$ .

a. What is the degree of  $P$ ?

b. Determine the maximum number of turning points on the graph of  $P$ .

c. Find the  $y$ -intercept of the graph of  $P$ .

d. Find the  $x$ -intercepts of the graph of  $P$ .

e. Determine whether the graph of  $P$  touches or crosses the  $x$ -axis at each  $x$ -intercept.

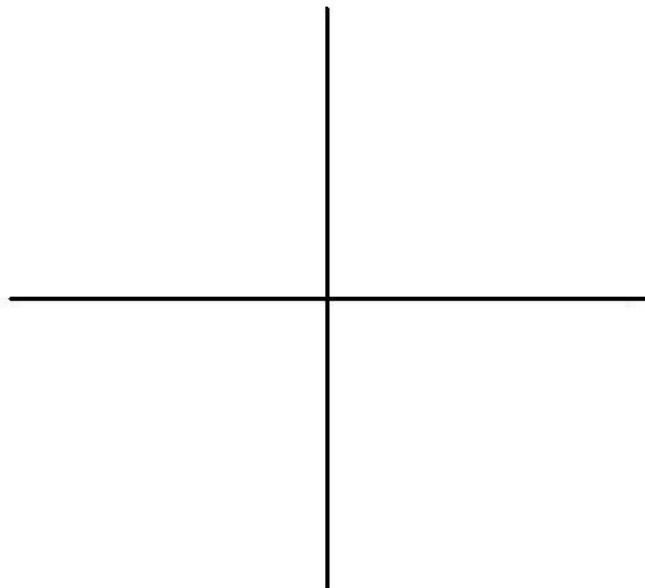
f. Determine the behavior of the graph of  $P$  near each  $x$ -intercept by completing the following:

label  $x$ -intercepts:



Interval determined by $x$ -intercepts			
Number chosen in interval			
Value of $P$			
Location of graph on interval			
Corresponding point on graph			

g. Put all of the information together to sketch the graph of  $P$ . Label at least 5 points on the graph.



2. Let  $P(x) = (x - 2)^2(x + 2)(x + 4)$

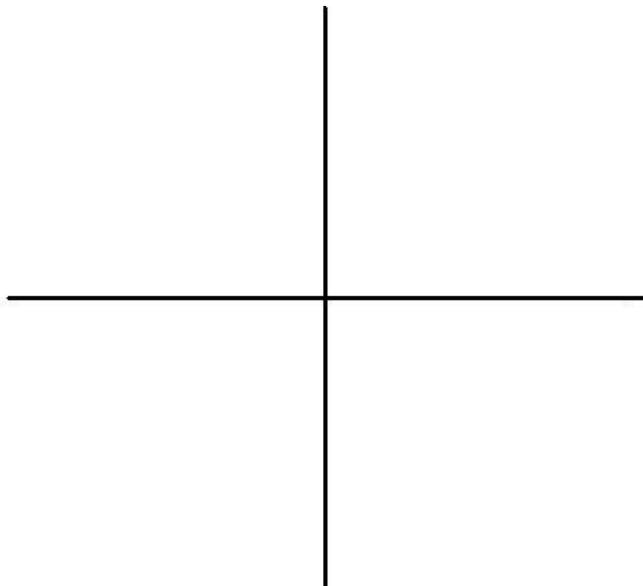
- a. What is the degree of  $P$ ?
- b. Determine the maximum number of turning points on the graph of  $P$ .
- c. Find the  $y$ -intercept of the graph of  $P$ .
- d. Find the  $x$ -intercepts of the graph of  $P$ .
- e. Determine whether the graph of  $P$  touches or crosses the  $x$ -axis at each  $x$ -intercept.
- f. Determine the behavior of the graph of  $P$  near each  $x$ -intercept by completing the following:

label  $x$ -intercepts:

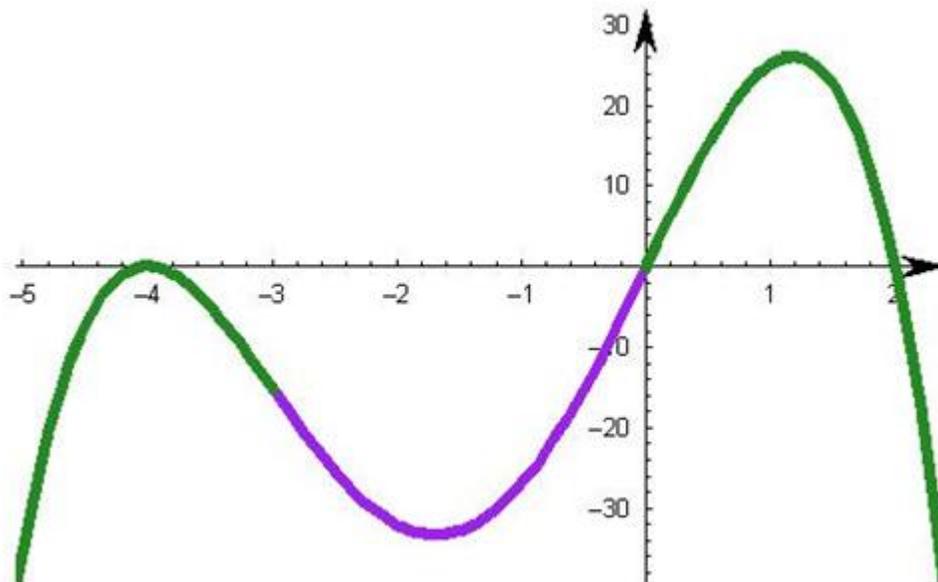


Interval determined by $x$ -intercepts				
Number chosen in interval				
Value of $P$				
Location of graph on interval				
Corresponding point on graph				

- g. Put all of the information together to sketch the graph of  $P$ . Label at least 7 points on the graph.



3. Given the graph of a polynomial function  $P(x)$  below, answer the following:



- a. What is the minimum degree of  $P$ ?
- b. What are the zeros of  $P$ ?
- c. Which zero of  $P$  has an even multiplicity?
- d. What is the sign of the leading coefficient of  $P$ ?
- e. Is the degree of  $P$  odd or even?
- f. Which of the following is most likely to be the formula for  $P$ ? (circle one)

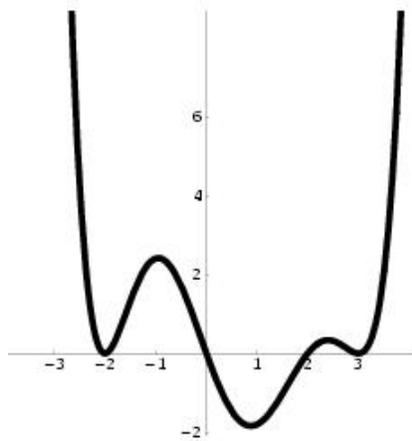
i)  $P(x) = x(x + 4)(x - 2)$

ii)  $P(x) = -x(x - 4)(x + 2)$

iii)  $P(x) = -x(x + 4)^2(x - 2)$

iv)  $P(x) = -x(x - 4)^2(x + 2)$

4. Given the graph of a polynomial function  $P(x)$  below, answer the following:



- a. What is the minimum degree of  $P$ ?
- b. What are the zeros of  $P$ ?
- c. Which zeros of  $P$  have an odd multiplicity?
- d. What is the sign of the leading coefficient of  $P$ ?
- e. Is the degree of  $P$  odd or even?
- f. Which of the following is most likely to be the formula for  $P$ ? (circle one)

i)  $P(x) = x(x + 2)(x - 2)(x - 3)$

ii)  $P(x) = x(x + 2)^2(x - 2)(x - 3)$

iii)  $P(x) = x(x + 2)^2(x - 2)(x - 3)^2$

iv)  $P(x) = x(x + 2)(x - 2)^2(x + 3)^2$

## 5.2, 5.3: Rational Functions

1. Let  $R(x) = \frac{2x + 4}{x - 1}$

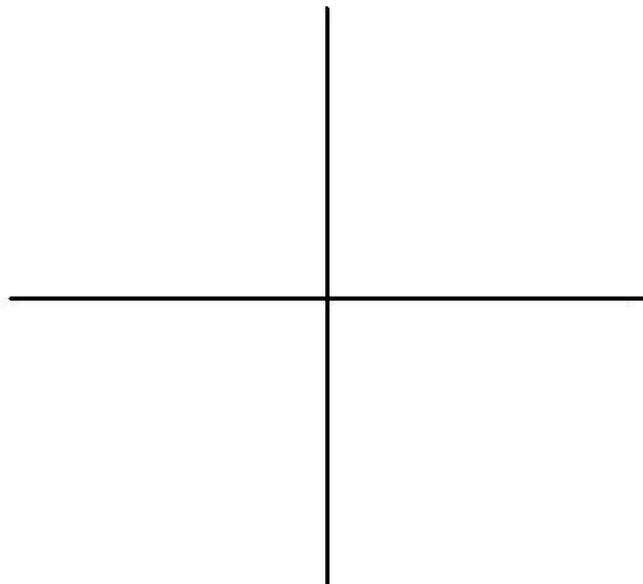
- Find the y-intercept of the graph of  $R$ .
- Find the x-intercept(s) of the graph of  $R$ .
- Find the equation(s) of the vertical asymptote(s) of the graph of  $R$ .
- Find the equation of the horizontal or oblique asymptote of the graph of  $R$ .

label zeros of numerator and denominator:



Interval determined by $x$ -intercepts			
Number chosen in interval			
Value of $R$			
Location of graph on interval			
Corresponding point on graph			

- Put all of the information together to sketch the graph of  $P$ . Label at least 5 points on the graph.



2. Let  $R(x) = \frac{x}{x^2 - 4}$

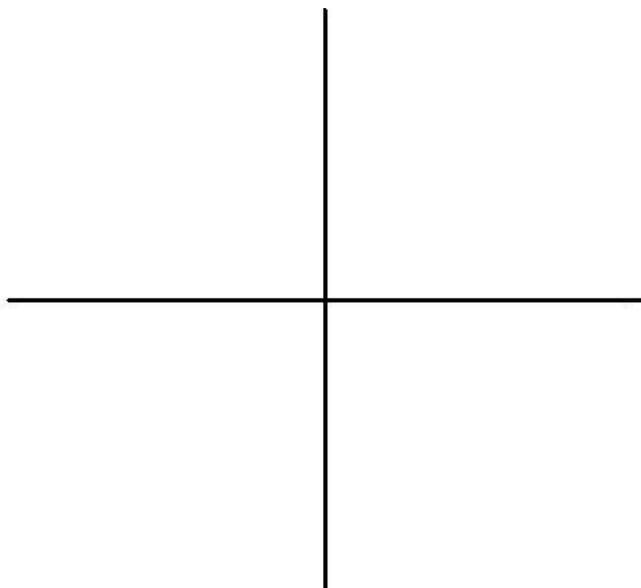
- a. Find the y-intercept of the graph of  $R$ .
- b. Find the x-intercept(s) of the graph of  $R$ .
- c. Find the equation(s) of the vertical asymptote(s) of the graph of  $R$ .
- d. Find the equation of the horizontal or oblique asymptote of the graph of  $R$ .

label zeros of numerator and denominator:



Interval determined by $x$ -intercepts				
Number chosen in interval				
Value of $P$				
Location of graph on interval				
Corresponding point on graph				

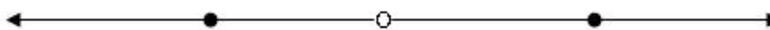
- g. Put all of the information together to sketch the graph of  $P$ . Label at least 5 points on the graph.



3. Let  $R(x) = \frac{x^2 - x - 12}{x + 1}$

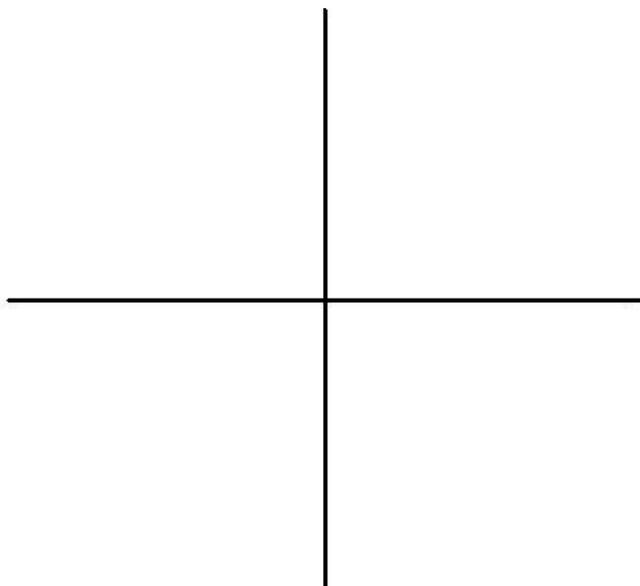
- a. Find the y-intercept of the graph of  $R$ .
- b. Find the x-intercept(s) of the graph of  $R$ .
- c. Find the equation(s) of the vertical asymptote(s) of the graph of  $R$ .
- d. Find the equation of the horizontal or oblique asymptote of the graph of  $R$ .

label zeros of numerator and denominator:



Interval determined by $x$ -intercepts				
Number chosen in interval				
Value of $P$				
Location of graph on interval				
Corresponding point on graph				

- g. Put all of the information together to sketch the graph of  $P$ . Label at least 5 points on the graph.



## 5.4: Polynomial and Rational Inequalities

Solve each of the following inequalities. Use a sign chart and show your work.

1.  $(x - 5)(x + 2) \leq 0$

2.  $x^3 + 2x^2 - 3x > 0$

$$3. \frac{3x - 5}{x + 2} \leq 2$$

$$4. \frac{5}{x - 9} \geq \frac{9}{5x - 5}$$

## 5.5: The Real Zeros of a Polynomial Function

1. Let  $P(x) = x^3 - 5x^2 - 61x - 55$

a. List all possible rational zeros based on the Rational Zeros Theorem.

b. Find all the zeros of  $P$ . (Show your synthetic division here.)

c. Use the zeros to factor  $P$  over the real numbers.

2. Let  $P(x) = 7x^4 + 6x^3 - 36x^2 - 30x + 5$

a. List all possible rational zeros based on the Rational Zeros Theorem.

b. Find all the zeros of  $P$ . (Show your synthetic division here.)

c. Use the zeros to factor  $P$  over the real numbers.

3. Let  $P(x) = x^4 + 6x^3 + 7x^2 - 12x - 18$

a. List all possible rational zeros based on the Rational Zeros Theorem.

b. Find all the zeros of  $P$ . (Show your synthetic division here.)

c. Use the zeros to factor  $P$  over the real numbers.

4. Let  $P(x) = 2x^3 - 7x^2 - 7x + 5$

a. List all possible rational zeros based on the Rational Zeros Theorem.

b. Find all the zeros of  $P$ . (Show your synthetic division here.)

c. Use the zeros to factor  $P$  over the real numbers.

## 5.6: Complex Zeros; Fundamental Theorem of Algebra

1. Construct a fourth degree polynomial having real coefficients with zeros  $-1$  and  $-3 - 2i$  where  $-1$  is a zero of multiplicity 2. (Express your answer in the form  $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ .)

2. Let  $P(x) = x^4 + 16x^2 + 15$

a. Find all the zeros of  $P$ . (Hint: no synthetic division is required)

b. Use the zeros to factor  $P$  over the real numbers.

3. Let  $P(x) = x^3 - 9x^2 + 36x - 54$

a. List all possible rational zeros based on the Rational Zeros Theorem.

b. Find all the zeros of  $P$ . (Show your synthetic division here.)

c. Use the zeros to express  $P$  as a product of linear factors.

4. Let  $P(x) = x^3 - 4x^2 + 4x - 16$ , where  $2i$  is a zero of  $P$ . Find the remaining zeros of  $P$ .

## 6.1: Composite Functions

1. If  $f(x) = 7\sqrt{x}$  and  $g(x) = 2x$ , find

a.  $(f \circ g)(8)$

b.  $(g \circ f)(1)$

c.  $(f \circ f)(49)$

d.  $(g \circ g)(-3)$

2. If  $f(x) = \frac{4}{x-7}$  and  $g(x) = \frac{1}{x}$ , find and simplify

a.  $(f \circ g)(x)$  and its domain

b.  $(f \circ f)(x)$  and its domain

c.  $(g \circ g)(x)$  and its domain

3. If  $f(x) = x^2 + 3$  and  $g(x) = \sqrt{x-7}$ , find and simplify

a.  $(f \circ g)(x)$  and its domain

b.  $(g \circ f)(x)$  and its domain

## 6.2: Inverse Functions

1. Consider the function  $f(x) = x^2 + 3$  where  $x \geq 0$ . Note  $f$  is one-to-one.

a. Find  $f^{-1}(x)$ .

b. Find the domain and range of  $f$ .

c. Find the domain and range of  $f^{-1}$ .

d. Sketch the graphs of  $f$ ,  $f^{-1}$ , and  $y = x$  on one set of axes.

2. The function  $f(x) = \frac{8x + 2}{9x - 7}$  is one-to-one.

a. Find  $f^{-1}(x)$ .

b. Find the domain of  $f$ .

c. Find the range of  $f^{-1}$ .

3. The function  $f(x) = \frac{x^2 - 3}{5x^2}$ , where  $x > 0$ , is one-to-one.

a. Find  $f^{-1}(x)$ .

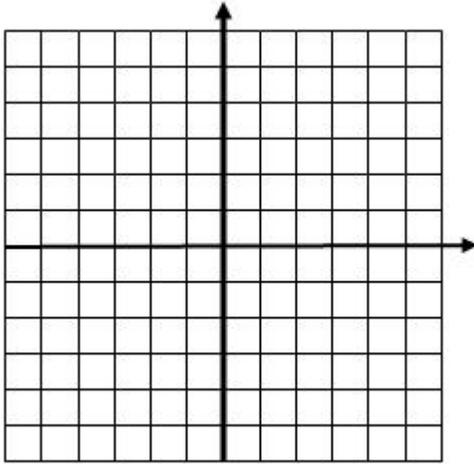
b. Find the domain of  $f$ .

c. Find the range of  $f^{-1}$ .

### 6.3: Exponential Functions

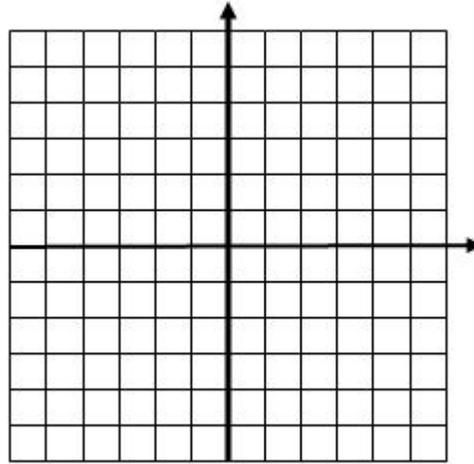
Problems 1-3: Graph each of the following functions using the techniques of shifting, compressing, stretching, and/or reflecting. Start by graphing a basic function and then show each stage of the transformation. Show the horizontal asymptote at each stage. Label at least three points on the final graph including the  $y$ -intercept.

1.  $f(x) = 2 - e^{-x}$



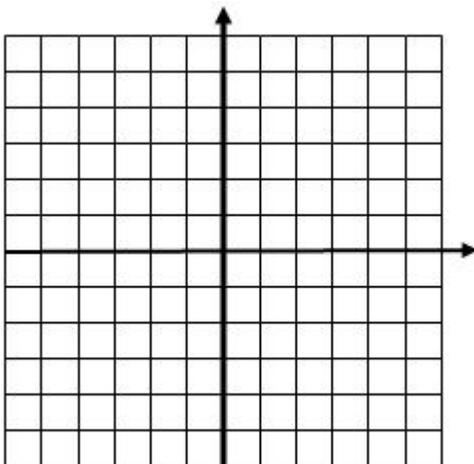
basic function:

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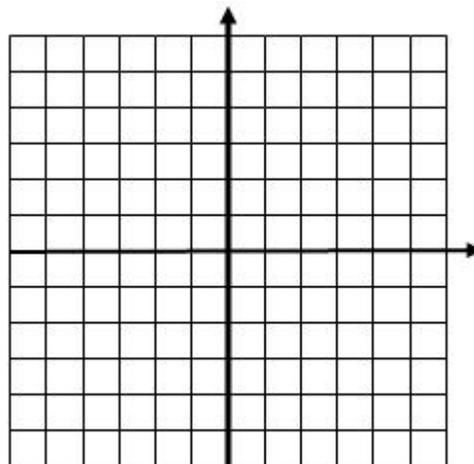
after 1st transformation:

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after 2nd transformation:

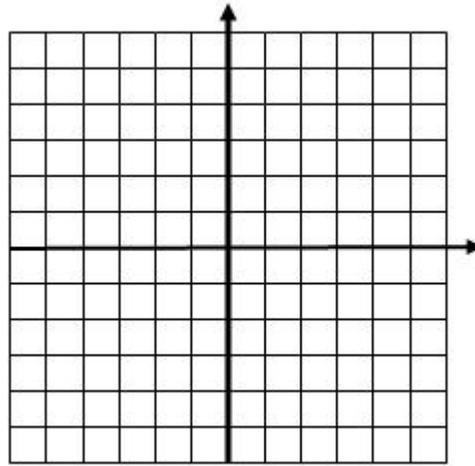
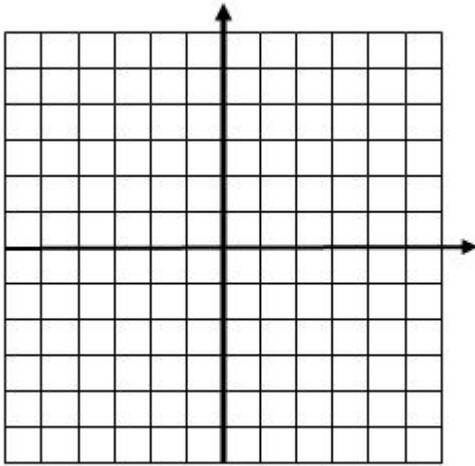
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after 3rd transformation (final answer):

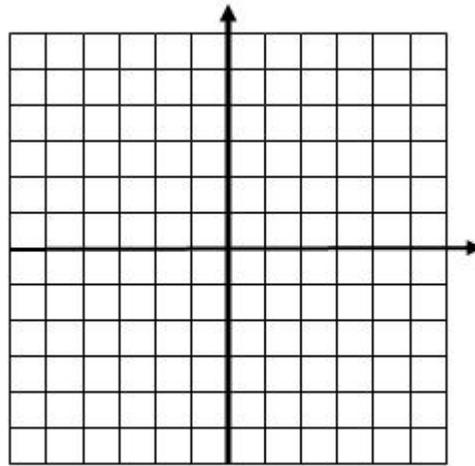
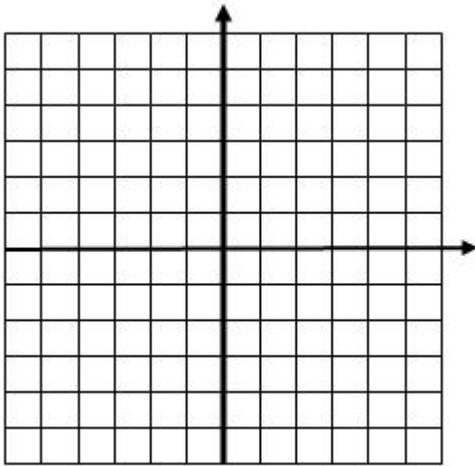
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2.  $f(x) = -2^{x+1} + 3$



basic function:

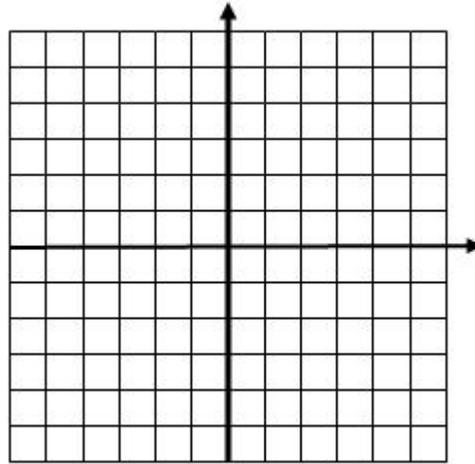
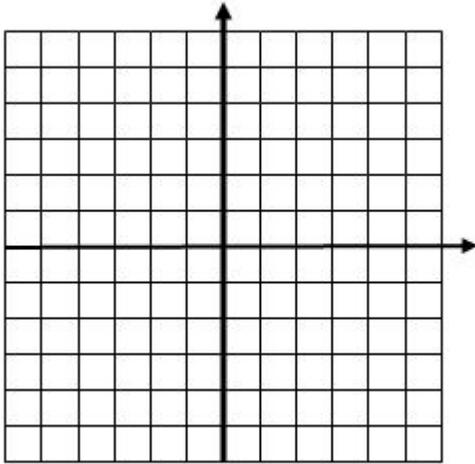
after 1st transformation:



after 2nd transformation:

after 3rd transformation (final answer):

3.  $f(x) = 3\left(\frac{1}{2}\right)^x - 1$



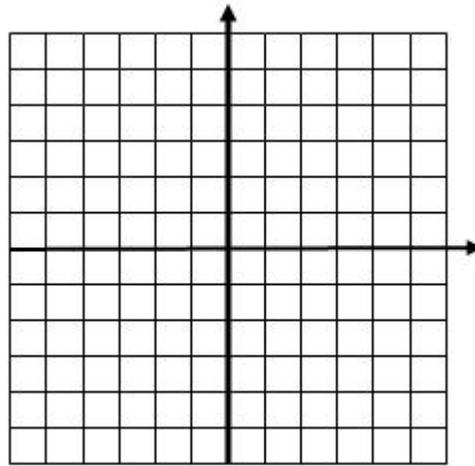
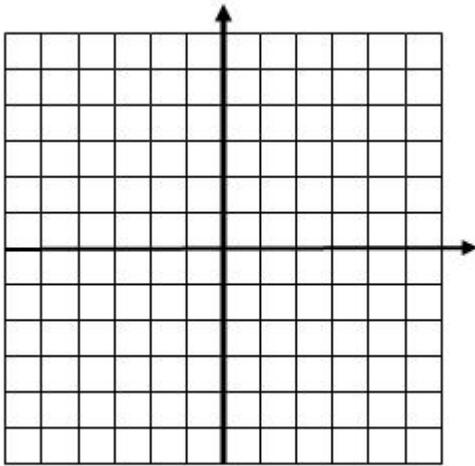
basic function:

after 1st transformation:

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after 2nd transformation:

after 3rd transformation (final answer):

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4. Solve the following equations for  $x$ .

a.  $3^{2x-1} = 9$

b.  $5^{x^2+8} = 125^{2x}$

c.  $e^{4x-9} = \frac{1}{e}$

d.  $(e^4)^x \cdot e^{x^2} = e^{12}$

## 6.4: Logarithmic Functions

Problems 1-6: Find the value of each expression without using a calculator.

1.  $\log_2 32$

2.  $\log_3 1$

3.  $\log_5 \left( \frac{1}{25} \right)$

4.  $\log_{\frac{1}{2}} 8$

5.  $\ln \sqrt[3]{e}$

6.  $\log_{25} 5$

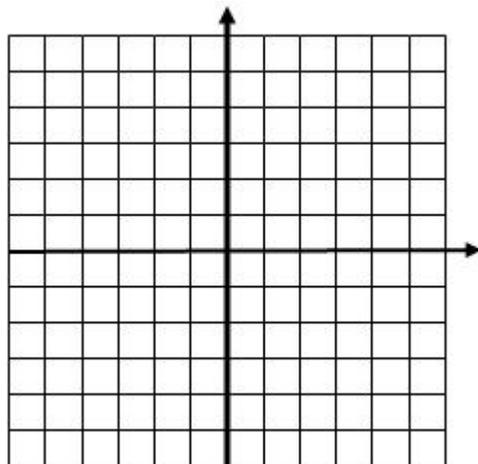
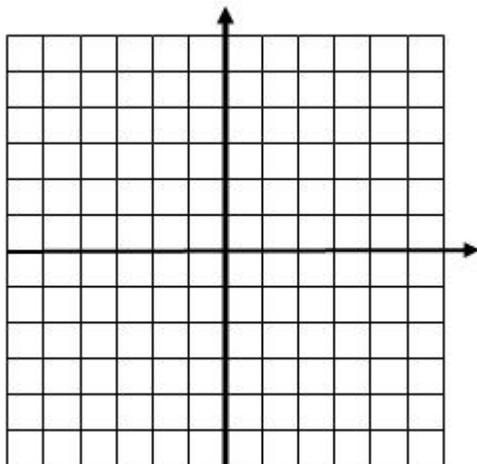
Problems 7 and 8: Find the domain of each function.

7.  $y = 5 - 2 \ln \left( \frac{x}{3} - 4 \right)$

8.  $y = \log_2 \left( \frac{2}{x-5} \right)$

Problems 9 and 10: Graph each of the following functions using the techniques of shifting, compressing, stretching, and/or reflecting. Start by graphing a basic function and then show each stage of the transformation. Show the vertical asymptote at each stage. Label at least three points on the final graph including the  $x$ -intercept.

9.  $f(x) = 1 - \ln(x - 2)$



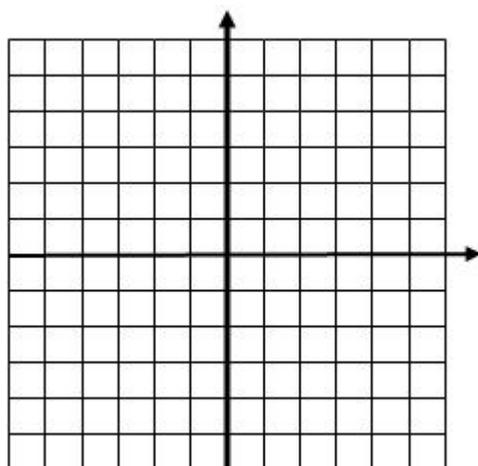
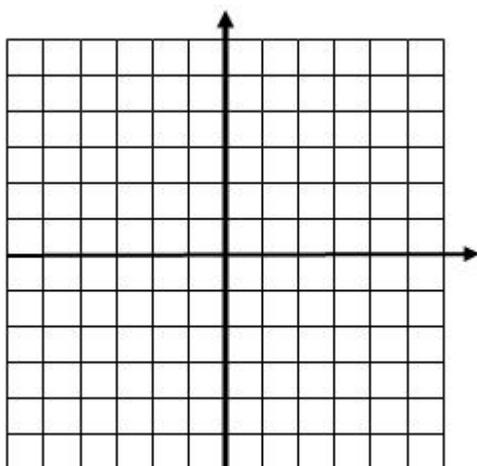
basic function:

after 1st transformation:

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after 2nd transformation:

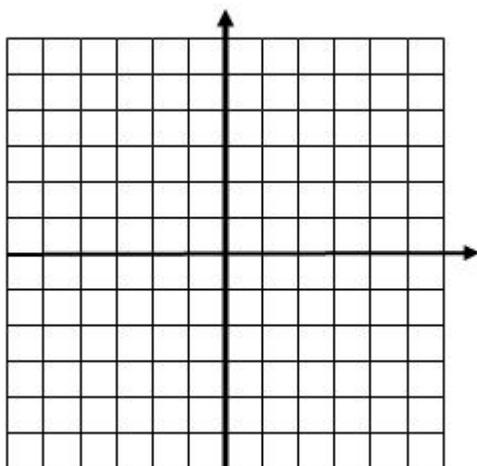
after 3rd transformation (final answer):

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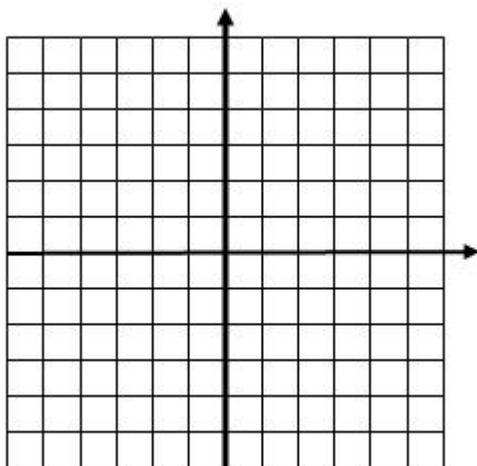
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10.  $f(x) = \log_2(-x + 1) - 1$



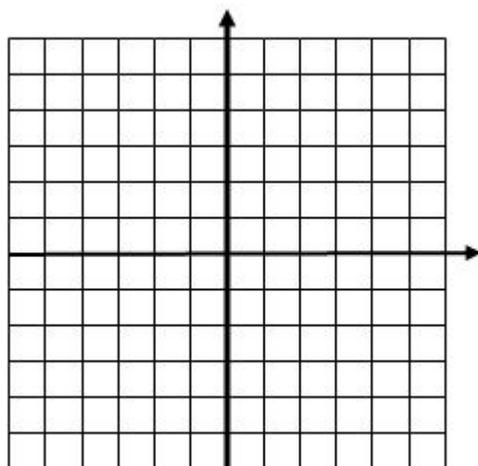
basic function:

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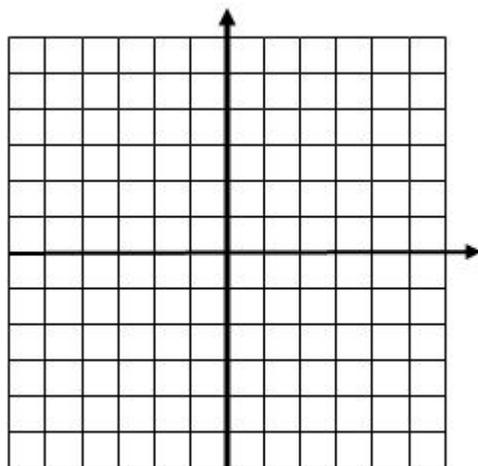
after 2nd transformation:

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after 1st transformation:

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after 3rd transformation (final answer):

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Problems 11-13: Solve each equation.

11.  $\log_4(3x - 5) = 2$

12.  $\log_2(x^2 + 3) = 4$

13.  $\log_5(x^2 + 3x + 4) = 2$

## 6.5: Properties of Logarithms

Problems 1-3: Find the exact value of the expression.

1.  $\log_2 160 - \log_2 5$

2.  $\log_{12} 9 + \log_{12} 16$

3.  $\log_3 100 - \log_3 18 - \log_3 50$

Problems 4 and 5: Write as sums and differences of logarithms. Express powers as factors.

4.  $\log \left( \frac{a^2}{b^4 \sqrt{c}} \right)$

5.  $\ln \left[ \frac{x^2 - x - 20}{(x + 7)^4} \right]^{\frac{1}{3}}$

Problems 6-8: Express as a single logarithm and simplify if possible.

6.  $4 \log x - \frac{1}{3} \log(x^2 + 1) + 2 \log(x - 1)$

7.  $18 \log_2 \sqrt{3x - 1} - \log_2 \left( \frac{9}{x} \right) + \log_2 9$

8.  $4 \log_4(2x) - \log_4(x^5) + \log_4(4x)$

## 6.6: Logarithmic and Exponential Equations

Solve each of the following equations.

1.  $9^x = 7$

2.  $2(8^{7x}) = 5$

3.  $\pi^{1-9x} = e^{8x}$

4.  $3^{1-6x} = 4^x$

5.  $\log_2(x^2 - x - 2) = 2$

6.  $\log_2 3 + \log_2 x = \log_2 5 + \log_2(x - 2)$

$$7. \log_2(x - 2) = 4 - \log_2(x + 4)$$

$$8. \log_{\frac{1}{6}}(x^2 + x) - \log_{\frac{1}{6}}(x^2 - x) = -1$$

## 6.8: Exponential Growth and Decay Models

1. A culture of bacteria obeys the law of uninhibited growth. If 500 bacteria are present initially and there are 600 after 1 hour,

a. Find a function,  $N(t)$ , that represents the number of bacteria after  $t$  hours.

b. How long does it take until the number of bacteria has increased by 40%?

2. Salt ( $\text{NaCl}$ ) decomposes in water into sodium ( $\text{Na}^+$ ) and chloride ( $\text{Cl}^-$ ) ions according to the law of uninhibited decay. If the initial amount of salt is 25 kilograms and after 10 hours, 15 kilograms of salt are left,

a. Find a function,  $A(t)$ , that represents the amount of salt (in kilograms) remaining after  $t$  hours.

b. How long does it take until half of the initial amount of salt remains?

3. The bones of a prehistoric man found in the desert of New Mexico contain approximately 5% of the original amount of carbon 14. If the half-life of carbon 14 is 5600 years, how long ago did the man die?

4. A population of bacteria obeys the law of uninhibited growth. If the population quadruples every day, how long does it take to triple?

## 2.4: Circles

1. Find both the standard and general form of the equation of a circle with radius 3 and center  $(-5, 2)$ .

2. Find the center and radius of the circle represented by  $x^2 + y^2 - 4x + 10y + 13 = 0$ .

3. Find the center and radius of the circle represented by  $2x^2 + 2y^2 - 12x + 8y - 24 = 0$ .

4. Find the standard form of a circle with endpoints of a diameter at  $(4, 3)$  and  $(0, 1)$ .

## 7.2: The Parabola

1. Find the equation of the parabola with vertex at  $(6, -2)$  and focus at  $(4, -2)$  and sketch its graph.

2. For the parabola defined by the equation  $x^2 + 6x - 4y + 1 = 0$ , determine the vertex, focus, and directrix and sketch the graph.

3. A parabolic reflector (paraboloid of revolution) is used by TV crews at football games to pick up the referee's announcements, quarterback signals, and so on. A microphone is placed at the focus of the parabola. If a certain reflector is 6 feet wide and 1.5 feet deep, where should the microphone be placed?

### 7.3: The Ellipse

1. Find the equation of the ellipse with vertices at  $(\pm 4, 0)$  and  $y$ -intercepts at  $(0, \pm 1)$  and sketch its graph.

2. For the ellipse defined by the equation  $x^2 + 9y^2 + 6x - 18y + 9 = 0$ , determine the center, vertices, and foci and sketch the graph.

3. A bridge is to be built in the shape of a semielliptical arch and is to have a span of 100 feet. The height of the arch, at a distance of 40 feet from the center, is to be 10 feet. Find the height of the arch at its center.

## 7.4: The Hyperbola

1. Find the equation of the hyperbola with vertices at  $(\pm 2, 0)$  and  $y$ -intercepts at  $(\pm 9, 0)$  and sketch its graph.

2. For the hyperbola defined by the equation  $x^2 - 4y^2 - 8x - 16y - 16 = 0$ , determine the center, vertices, foci, transverse axis, asymptotes and sketch the graph.

## 8.1: Systems of Linear Equations: Substitution and Elimination

1. Solve the system of equations:

$$\begin{cases} \frac{1}{3}x - \frac{3}{2}y = -5 \\ \frac{3}{4}x + \frac{1}{3}y = 11 \end{cases}$$

2. Solve the system of equations:

$$\begin{cases} x - y + 3z = 4 \\ x + 2y - 2z = 10 \\ 3x - y + 5z = 14 \end{cases}$$

3. Find real numbers  $a$ ,  $b$ , and  $c$  so that the function  $y = ax^2 + bx + c$  contains the points  $(-1, 6)$ ,  $(2, 8)$ , and  $(0, 4)$ .

## 8.6: Systems of Nonlinear Equations

1. Solve the system of equations:

$$\begin{cases} x^2 - y = 2 \\ 2x - y = -1 \end{cases}$$

2. Solve the system of equations:

$$\begin{cases} 3x^2 + 2y = 26 \\ 5x^2 + 7y = 3 \end{cases}$$