CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

Multifractal Properties of EUV Intensity Fluctuations and
Implications for Impulsive Heating Mechanisms of the Solar
Corona

A thesis submitted in partial fulfillment of the requirements for the
degree of Master of Science in Physics

by

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Dedication

To my wonderful mother and sister. Thank you for all your support, love and motivation. To my fantastic soul mate that has kept me positive and has guided me through this marvelous journey.
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ABSTRACT

Multifractal Properties of EUV Intensity Fluctuations and Implications for Impulsive Heating Mechanisms of the Solar Corona

By

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Master of Science in Physics

Much work has focused on explaining the mechanism behind the heating of the solar corona. Due to instrument limitations in spatial resolution we cannot study the heating directly. Instead statistical analysis techniques have been applied to study characteristics of the solar emission. In this study we investigated the scaling properties of Extreme Ultraviolet (EUV) intensity fluctuations in an Active Region (AR) from images taken by the Solar Dynamics Observatory (SDO). Four separate regions were studied: the core, a weak emission zone and two distinct core loops. Two complementary methods were used in the analysis. First we calculated the probability distribution functions (PDF) of the increments and followed with the multifractal detrended fluctuation analysis (MF-DFA). The latter takes into account the non-stationary characteristics of the data and eliminates external trends to identify the long range correlations in the time series. Used together they provide characteristic "signatures" of the observed radiation. We distinguished between emission dominated by the corona or the transition region (TR) by considering the lags in pairs of EUV images. While noise is present in all EUV wavebands, it dominates in the
hotter channels. Therefore the cooler emission in the 171Å waveband is the more appropriate for these types of analysis. For all regions tested and for both positive and zero lag pixels, we found that at large temporal scales in the approximate fitting range of 15 - 45 min, that the observed time signals are anti-persistent. The pixels with zero lag associated with the TR emission, present stronger anti-correlation compared to those of coronal emission. The signals present varying degrees of multifractality which is a consequence of long-range temporal correlations. The emission in the 335Å waveband has the properties of a multifractal contaminated with noise. The intensity simulated via a phenomenological model for impulsive heating, and the averaged ohmic dissipation in a reduced MHD model for coronal loops, both exhibit multifractal properties and anti-persistence. For the cases studied the PDF of increments differs from those found in the observations. We foresee that the two analysis methods applied in tandem can be effectively used to optimize the parameters in models for coronal heating.
1.1 The Solar Physics problem

The Sun is composed of various layers with different plasma properties. This state of matter in which atoms are ionized is highly susceptible to magnetic fields. The core at the center of the Sun is the hottest (15 million Kelvin) and densest ($150\text{g/cm}^3$) region. Here the energy is generated through fusion reactions. The surrounding radiative and convective zones cool through radiation and turbulent convection respectively. At the visible surface, the photosphere, the temperature decreases to $\sim6000\text{K}$. It then reaches a minimum in the lower chromosphere and subsequently the temperature rises to millions of degrees Kelvin from the upper chromosphere, through the transition region (TR) to the tenuous corona (Grotian, 1939). The sharpest increase occurring at the Transition Region (TR) (Gary et al., 2007; Browning, 1991). Due to the high temperatures, the corona is predominately emitting in the EUV and X-ray range not visible to the human eye, only detected through specific spectral filters. Figure 1.1 shows the temperature and density as functions of height from 100 km above the photosphere to 10,000 km in the corona. If heat energy is transmitted through radiation it is expected that as you move further away from a heat source the temperature of the surrounding material would decrease. However the temperature structure of the solar atmosphere does not fit this picture. The steep temperature gradient is puzzling because there is no clear standard mechanism to explain it.

![Figure 1.1: Image taken from Gary et al., 2007. A log-log plot of the temperature and density variations versus the distance above the photosphere.](image)

1.2 What mechanism is maintaining the corona at such high temperatures?

Magnetic fields of varying strength and topology (open and closed) permeate the Sun. The convective turbulent motions continuously shuffle the magnetic field flux in the photo-
sphere. In one model for coronal heating it is expected that these motions produce Alfvén waves which propagate along the magnetic fields to the corona and dissipate heat into the surrounding plasma [Roberts, 2000; De Moortel and Browning, 2015; Asgari-Targhi et al., 2015]. In an alternative scenario the motions at the footpoints build up stress in the coronal magnetic field. This leads to the formation of current sheets which become the sites for impulsive energy release via magnetic field reconnection [Parker, 1972; Biskamp, 1986].

Flares are the result of large scale reconnection events, releasing as much as $10^{34}$ ergs per event, however, are too infrequent to be the dominant source for heating. Instead Parker [1988] suggested that the Sun was constantly heated by smaller more sporadic events that he termed “nanoflares”. These are eruptions defined by their characteristic energy release between $10^{23} - 10^{26}$ ergs per event well below that of a normal class flare $>10^{27}$ ergs [Parker, 1972]. A main challenge to study small impulsive events is that they occur at spatial scales below the resolution of the instrument. Dahlburg et al. [2016] has suggested that the relevant scale for reconnection is in the order of the ion (proton) inertial length, $d_i$, given by $d_i = \frac{c}{\omega_{pi}} \sim 23m$ where $c$ is the speed of light and the proton plasma frequency is given as $\omega_{pi} = \sqrt{4\pi n_i e^2/m_i}$ where the ion density $n_i \sim 10^8 cm^{-3}$, $e$ is the electron charge and $m_i$ is the proton mass. The value $d_i \sim 23m$ is much smaller than any pixel scale of current solar observations. At present the instrument with the highest resolution for coronal observations is the sounding rocket Hi-C sounding rocket Hi-C at $0^\circ.103$ per pixel or $\sim 75$ km [Cirtain et al., 2013].

While there is not a unifying picture which can explain coronal heating, at present there is consensus that impulsive heating as described in the nanoflare scenario plays an important role independent of the underlying detailed physical mechanisms [Klimchuk, 2006, 2014; De Moortel and Browning, 2015 and references therein]. The energy bursts may be the result of reconnection as described above [Parker, 1972, 1988], or possibly due to the dissipation of energy in magnetohydrodynamic (MHD) turbulence models [ie. Nigro et al., 2004; Reale et al., 2005; Rappazzo et al., 2008; Rappazzo and Parker, 2013; Asgari-Targhi and van Ballegooijen, 2012].

Coronal loops are arch like structures above the surface of the Sun which are visible because of the radiation emitted by the plasma within the magnetic field lines. It is expected that below the observational resolution the loop structure is formed by a multiplicity of magnetic field strands. In the standard nanoflare model based on magnetic reconnection, different outcomes follow from the different ways in which the sub-resolution strands are heated. In a high-frequency process the time between heating impulses is shorter than the cooling times of the strands, thus smoothing the changes over a period of time. In the case of low-frequency heating the time between bursts is longer than the cooling time of the structure, and the strands evolve individually to cooler temperatures [Cargill and Klimchuk, 1997; Tripathi et al., 2011; Winebarger et al., 2011]. A loop can contain a small number of strands that are all heated at about the same time or it can contain many strands all heated independently at random times. In both cases the simulations are identified are “nanoflare storms” with different properties [Klimchuk, 2009].
Because it is not possible to “see” the scales where the physical processes take place an approach to gain understanding consists on extracting characteristic “signatures” from the observations and comparing to the results obtained in the analysis of modeled data. Some progress has been made via statistical studies. For example number distributions of flare energies in X-ray emission have been found to exhibit a power law dependence, \( dN/dE = E^{-\alpha} \) where \( E \) is the flare energy and \( dN \) is the number of events within the energy interval, \((E, E + dE)\). For a given \( \alpha \) greater than 2, small-scale events in the nanoflares energy range dominate the heating process [Hudson, 1991]. Other work has shown that the intensity distributions of the quiet Sun, chromosphere, TR and lower corona are log-normal [Pauluhn et al., 2000, 2001]. This has been used to optimize a nanoflare phenomenological model for coronal radiation (Pauluhn & Solanki 2007). In a different approach [Viall and Klimchuk, 2011], using the enthalpy-based thermal evolution of loops (EBTEL) 0-dimensional hydrodynamic model [Klimchuk et al., 2008], predicted that the corona emission in the EUV follows a cooling pattern with radiation in the hotter wavelengths leading and the cooler wavebands following in sequential order. By calculating the lags between pairs of contemporaneous coronal intensity observations in six EUV channels they there were able to confirm this prediction [Viall and Klimchuk, 2011, 2012]. In later work Viall and Klimchuk [2015] show that the TR responds differently to an impulsive heating model with all the EUV channels peaking at the same time and leading to zero lags when comparing image pairs.

In previous work, Cadavid et al. [2014] showed that EUV radiation from the apex of active region (AR) coronal loops presented emission patterns, temperature and density relations compatible with a nanoflare storm model. They also found power spectra of the EUV radiation obeyed power laws with a scaling exponent \( \beta > 1 \), suggesting non-stationary behavior [Mandelbrot and Van ness, 1968]. This opened the question of whether the signals had properties akin to fractional Brownian motion (fBm) characterized by a single scaling exponent \( 1 < \beta < 3 \), (a mono fractal) or whether they were more complex with multiscaling properties (a multifractal).

In this thesis we present the results of this investigation using two complementary methods: the calculation of the probability distribution function (PDF) of increments [Budaev, 2005] and the multifractal detrended fluctuation analysis (MF-DFA). The latter is used to find the true scaling properties of a signal, identifying long-term correlations in noisy and non-stationary time series after accounting for external trends [Kantelhardt, 2008]. For the data we used observations of EUV emission obtained with the Atmospheric Imaging Assembly on board of the Solar Dynamics Observatory (SDO) [Pesnell et al., 2012; Lemen et al., 2012]. The study focused on the 171Å and 335Å as examples of cool and hot emission respectively. We selected four physical regions AR core, weak emission and two core coronal loops, and separated the pixels based on whether they corresponded to coronal or TR emission. We found that the 171Å signals, with a high signal to noise ratio (SNR) are clearly multifractal as a consequence of long term correlations and not due to heavy tails in the distribution of the intensity values. The TR signals have stronger anti-correlation (anti-persistence) than those in the corona. The weaker 335Å coronal emission (lower SNR) can be described in terms of a multifractal “corrupted” by noise. Similar analysis was applied
to the simulated intensities using the aforementioned model for impulsive coronal heating [Pauluhn and Solanki, 2006] plus added noise, as well as to the outputs from a reduced magnetohydrodynamic (RMHD) turbulence model [Rappazzo et al., 2008, 2013]. Here we report to what extent the models have the potential of explaining the characteristics signatures in the observations. We propose that the analysis methods introduced in the present work, which provide strong constraints, can be used to systematically test models for impulsive heating.
Chapter 2
Observations and Analysis

2.1 Data Preparation

2.1.1 AIA/SDO data

Observations from AIA/SDO with their continuous coverage and high temporal cadence are ideal for the type of analysis proposed. Images are available in six EUV wavebands (171, 193, 211, 335, 94, 131Å) primary centered on iron lines (Fe IX and Fe X, XII, XIV, XVI, X & XVIII, Fe VIII & Fe XX). The images run through a series of processing stages where they are corrected for cosmic ray interaction or bad pixels and are adjusted for geometric corrections. The various levels of cleaning can be found at the AIA processing website\(^1\). The original data are 4096x4096 pixel images with spatial and temporal resolution of 1” and cadence 12s, respectively. Additional information on the instrumentation onboard SDO can be found in Lemen et al. [2012] and Boerner et al. [2014].

In the present work, we present the results for two spectral lines, the 171Å (in the range 0.63-1.1 MK) and 335Å (~2.7 MK) examples of cooler and hotter emission respectively [Petkaki et al., 2012]. The 171Å waveband was chosen for its high intensity count that minimizes noise effects in the analysis while omitting the 193Å and 211Å channels with statistically similar features. The 335Å was chosen as a representative of the hotter channels, being the one with the relative lowest signal to noise ratio. We use data for NOAA AR11250 during its first passage as it transits the solar meridian at S27. For the original study on the core loop emission, this non-flaring AR was chosen because of its high variability in the intensity fluctuations in 94Å waveband which suggested impulsive heating. In the present study, in addition two core loops, as shown in Figure 2.1 we also include two other emission regions: the core, situated between the two sunspots located using the magnetic images from the Helioseismic and Magnetic Imager (HMI) instrument on SDO [Scherrer et al., 2012], see Figure 2.2, and a peripheral weak emission zone chosen for steady activity associated with basal heating. The loops considered are displayed in inverted gray scale in the hotter 94Å band during peak intensity periods where they appear more visible (Figure 2.3).

Once downloaded, from the full disk images a 512x512 section is cut that included the entire AR zone. No flares were reported by the online Solar Monitor\(^2\) in that region during the length of the observation. We have analyzed 1350 temporal images (270 min) starting at UT 12:02 to 16:32 on 2011 July 13. In order to have continuous observations of the AR location, we first de-rotated the images to align them using IDL.

\(^1\)http://jsoc.stanford.edu/doc/data/aia_test/SDOD0045_v10_AIA_plan_for-producing_and_distri_AIADataP_gPlanv1-6.pdf

\(^2\)http://www.solarmonitor.org/index.php?date=20110713&region=11250&indexnum=1

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Figure 2.1: NOAA AR11250, in (left) 171Å with designated core and weak emission zones and (right) 335Å. Each image is a temporal average image from 8 evenly separated frames within the observation time frame.

Figure 2.2: Magnetic image of the AR11250 showing the position of the core, situated between two sunspots where the white represents positive polarity and black, negative polarity on the solar surface.
2.1.2 Data cleaning

Additional steps were taken to exclude pixels with large fluctuations that may correspond to large scale events such as microflares or transient brightenings. We also excluded slow varying signals that do not exhibit intermittent fluctuations characteristic of nanoflare heating. To examine the light curves for large scale brightenings we used the technique by Terzo et al. [2011] where any pixel value larger than a given percentage of the best fit line of the data at that point disqualifies the entire light curve and is omitted from further analysis. It was estimated that 200% is a reasonable intensity fluctuation from the best fit line. To remove the slow varying time series we set a criterion for a minimum number of crossings defined by the standard deviation of a binomial distribution, i.e. $\sqrt{m - 1}/2$ where $m$ is the number of data points in the series [Terzo et al., 2011; Jess et al., 2014]. For all observations, $m = 1350$ pixels making the minimum number of crossings for each series 19. The IDL program, zero_cross.pro was used for this analysis. In the core region for the 171Å emission these criteria omitted 485 pixels (10 pixels due to slow varying behavior) or roughly 20.2% of the total pixels at the core (see Figure 2.4). For the weak emission zone we found only 169 pixels (13 pixels due to slow varying behavior) or $\sim 7.5\%$ of the total pixels. The pixels identified by their slightly large excursion from the best fit line were not large spikes as in the emission in the core, therefore we decided to forego data cleaning for the weak emission. For loops 1 and 2 a total of 36% and 24% were excluded respectively.

2.1.3 Cross-correlation computation

Following Viall and Klimchuk [2012], we compute the cross-correlation between the 335-171Å and the 211-171Å pairs. The calculation is performed using the predefined routine, c_correlate.pro code in IDL. For negative temporal offsets at lag $L$, the correlation function, $P(L)$, is defined as
Figure 2.4: An image of the core in the 171Å waveband (see Figure 2.1). The colorbar values are intensity values in $DN \, s^{-1}$. Pixels in black have been omitted due to slow variability or high pixel fluctuation caused by large scale brightenings beyond those expected from nanoflaring.

The result is an image containing the value defined by the maximum correlation value at each pixel. The lag value can be positive, negative or zero (see Figure 2.5). A zero lag is defined for values at the time resolution scale ($-12 \, sec < L < 12 \, sec$). Using this technique we identified as corresponding to coronal emission the pixels which presented positive lags in both the 335-171Å and the 211-171Å pairs. Similarly the common pixels with zero lag were associated with TR emission.

\[ P(L) = \frac{\sum_{k=0}^{N-|L|-1} (x_{k+|L|} - \bar{x})(y_k - \bar{y})}{\sqrt{\sum_{k=0}^{N-1} (x_k - \bar{x})^2 \sum_{k=0}^{N-1} (y_k - \bar{y})^2}} \quad (2.1) \]

the corresponding formula for positive offsets is

\[ P(L) = \frac{\sum_{k=0}^{N-L-1} (x_{k} - \bar{x})(y_{k+L} - \bar{y})}{\sqrt{\sum_{k=0}^{N-1} (x_k - \bar{x})^2 \sum_{k=0}^{N-1} (y_k - \bar{y})^2}} \quad (2.2) \]

where $N$ is the number of points in the time series and, $x$ and $y$ are intensities in the two wavebands.

The result is an image containing the value defined by the maximum correlation value at each pixel. The lag value can be positive, negative or zero (see Figure 2.5). A zero lag is defined for values at the time resolution scale ($-12 \, sec < L < 12 \, sec$). Using this technique we identified as corresponding to coronal emission the pixels which presented positive lags in both the 335-171Å and the 211-171Å pairs. Similarly the common pixels with zero lag were associated with TR emission.

2.2 Distribution of Increments

Power spectra for loop apex emission was found to have a scaling exponent $\beta > 1$. This opened the possibility that the signal could correspond to fractional Brownian motion (fBm), characterized by a single scaling exponent (a monofractal). While traditional Brownian motion has uncorrelated increments, fBm has increments with positive or negative correlation [Mandelbrot and Van Ness, 1968]. In order to test if indeed we were dealing with a monofractal or with more complex signals with a spectrum of scaling exponents.
Figure 2.5: Lag maps for the 211-171Å (left) and 335-171Å (right) pairs. We use a maximum temporal offset, $L$, of $\pm1$ hour for the 4.5 hour observation. The core and weak emission locations are denoted by the black rectangles. The colorbar values represent the temporal offsets in seconds.

(a multifractal) we investigated the probability distribution function (PDF) of increments over a range of scales. For a textbook monofractal signal the increments for all scales are normally distributed. This self-similarity relationship between scales can be described in terms of the Hurst exponent, $H$ [Budaev, 2005]. This quantity was originally introduced by Harold Edwin Hurst (1951) in the field of hydrology while studying the water levels of the Nile River. Occasionally referred to as the “index of dependence”, it provides a measure of how values in a time series are correlated in time. The Hurst exponent has the range $0 < H < 1$, where $H = 0.5$ indicates an uncorrelated series such as Gaussian white noise, $H > 0.5$ indicates a long-range correlated (“persistent”) series, and $H < 0.5$ an anti-persistent series [Hurst, 1951]. For example, in a persistent time series, i.e. $H = 0.7$, positive fluctuations are likely to be followed by future positive fluctuations and negative fluctuations by negative fluctuations.

Following Budaev [2005], a mono-fractal time series, $x(t)$, is said to be a self-similar monofractal if the probability density function is related as

$$P_l(\delta_l x) = x^H P_\lambda(l^H \delta_x)$$  \hspace{1cm} (2.3)

Where $\lambda > 0$ and $\delta_l x$ is the increment value at scale $l$ defined as

$$y(k) = \delta_l x = x(k+l) - x(k)$$  \hspace{1cm} (2.4)

For a multifractal there exist a spectrum of exponents that define the local self-similarity making the relationship between scales much more complex [Budaev et al., 2006; Kiyani et al., 2007]. In this thesis we take an empirical approach and calculate the PDFs of increments for a generated multifractal. These together with those of the simulated monofractal
are used as references for the data.

To better compare the increment distributions, we define the scaled PDF as

$$ P_s(y) = \frac{1}{\sigma} P \left( \frac{y - \langle y \rangle}{\sigma} \right) $$

(2.5)

where $P_s$ is the histogram for the defined variable, $y$ are the values of increments at scale $l$, $\langle y \rangle$ is the mean and $\sigma$ is the standard deviation. It is recommended that the increment size, \(l\), between intensity values does not exceed \(1/3\) of the size of the signal for more reliable results \cite{Budaev2006}.

The monofractal time series were obtained using a fractional Brownian motion simulation in MATLAB, fbm1d.m \cite{Kroese2014}. For both the monofractal and multifractal examples We generated 2000 realizations with 1350 pixels in temporal length. The multifractal signals were generated via binomial multiplicative cascade (BMC) due to its relatively simple application. The BMC method begins at unit length that is redistributed at every iteration based on a chosen distribution factor, \(d\) where \(0 < d < 1\), such that \(d\) is multiplied to one side and \((1 - d)\) to the other, preserving the original measure, continued for the desired number of iterations \cite{P2003}.

The distributions are plotted on a log-linear plot (Figure 2.6) where a Gaussian $P_s$ corresponds to a parabola.

$$ P_s = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(\frac{(x-\mu)^2}{2\sigma^2}\right) $$

(2.6)

Where $\mu$ is the mean and $\sigma$ is the standard deviation.

The logarithmic of $P_s$ is given by,

$$ \log(P_s) = \log \left( \frac{1}{\sigma \sqrt{2\pi}} \exp\left(\frac{(x-\mu)^2}{2\sigma^2}\right) \right) $$

(2.7)

Simplifying the right hand side,

$$ \log(P_s) = -a(x - \mu)^2 + \log(b) $$

(2.8)

The equation becomes a parabola with vertex \((\mu, \log(b))\) where $b = 1/(\sigma \sqrt{2\pi})$, and amplitude $a = 1/(2\sigma^2)$

### 2.2.1 PDFs: Mono-fractal versus Multi-fractal

For a monofractal (Figure 2.6, Left), the shape and statistical properties of the increments at all scales closely follow a Gaussian distribution where for a normal distribution the skewness (third moment) and kurtosis (fourth moment) are zero (skewness=kurtosis=0). The shape remains parabolic and is unaffected by scale size with kurtosis values of zero.
In the case of the multifractal (Figure 2.6, Right), multi-scaling is apparent for the PDFs at different scales which deviate from the self-similar monofractal case. This can be quantified in terms of the variations in skewness and kurtosis. At small scales, the PDFs are characterized by “fat tails” and appear tent-like with a high kurtosis value. With increasing increment size the profiles evolve to a “quasi-Gaussian” shape and kurtosis converging to zero.

2.2.2 PDFs: Core

We calculate the PDFs at scales (2, 8, 32, 128, 384) pixels corresponding to (0.4, 1.6, 6.4, 25.6 and 76.8) min. For both the coronal and TR cases in the core we find some general trends (Figure 2.7). At the smaller scales in the 171Å waveband the shape of the PDFs appears tent-like with tails resembling those of multifractal PDFs. The 335Å signals also exhibit fat tails however appear to have more rounded tops. A possible explanation of this effect can be due to noise being more prominent in the hotter channels. The average intensity at the core in the 335Å is roughly 276 $DN s^{-1}$, compared to the cooler 171Å, with an average intensity of 1,166 $DN s^{-1}$. At the largest scale used, 76.8 min, the distributions become distorted showing an upper limit for the increment given the length of the time series. In all wavebands, the kurtosis at small scales starts at high values that decrease with increasing temporal scale, becoming more Gaussian-like, as in the simulated multifractal analyzed in Figure 2.6. The kurtosis values from the smallest to the largest scale: 171Å corona (positive lag) (18.2, 5.63, 2.56, 1.68, 1.18), 171Å TR (zero lag) (9.24, 4.73, 3.91, 1.43, -0.06) and 33Å positive lag values (34.4, 27.83, 16.64, 4.95, 1.35). Both the shapes of the PDFs as well as the kurtosis variation are strong indicators of multifractality.

2.2.3 PDFs: Weak Emission

For the weak emission for PDFs corresponding to positive lag values, displayed in Figure 2.8, the noise effect is more prominent in both wavebands because the intensity values...
Figure 2.7: The log-linear plot of PDFs from increments at the core (see Figure 2.1). The PDFs have been vertically shifted for clarity. Left to right: 171Å (positive lag), 171Å (zero lag) and 335Å (positive lag) increments. Scales plotted from top to bottom 0.4, 1.6, 6.4, 25.6 and 76.8 min.

are lower compared to those seen at the core. The average intensity in the weak emission region is: \(\sim 192 \text{ DN s}^{-1}\) (171Å) and \(\sim 15 \text{ DN s}^{-1}\) (335Å). For the 171Å waveband (top left) the kurtosis values are (6.9, 4.72, 3.13, 1.66, 0.21), starting higher at small scales and tending to zero for large scales. In the 335Å signal (top right), the PDFs are visually similar in shape. In this case the signal is dominated by noise leading to a common “quasi-Gaussian” profile across scales. The kurtosis has a small dependence on scale with values: (2.11, 2.09, 1.90, 1.66, 1.07) from top to bottom in the figure.

2.2.4 PDFs: Loops

Figure 2.8, bottom row, displays the PDFs of increments for 171Å coronal emission (positive lag) for both loops. The PDFs appear fairly peaked at small scales, as seen in the core, however the distributions at the largest scales become highly distorted. Given that the pixel count for the loops is much lower than that of the core and weak emission areas, the statistics at this size are poor. Nevertheless we see the shape is quasi-Gaussian. Again from small to the large scales the kurtosis are: for Loop 1 (bottom-left) (11.85, 4.39, 2.10, 1.28, 0.78) and for Loop 2 (bottom right) (19.90, 8.94, 3.61, 0.40, -0.55, -0.07). The skewness hovers around zero in both cases.
Figure 2.8: The log-linear plot of PDFs from increments at the peripheral weak emission (see Figure 2.1). The PDFs have been vertically shifted for clarity. Top, Left: 171Å (positive lag), right: 335Å (positive lag). Bottom, Left: Loop 1, 171Å (positive lag), right: Loop 2, 171Å (positive lag). Scales plotted from top to bottom 0.4, 1.6, 6.4, 25.6 min and 76.8 min except for Loop 2 where last increment is 38.4 min instead of 76.8 min
Due to the evidence of multiscaling in time series of the core and the weak emission areas identified through the PDFs, we continued to investigate the fractal nature of the signals using the MF-DFA. The MF-DFA is an extension of the DFA method developed to provide a full description of the fractal system through the range of moments, $q$. The DFA was initially introduced by Peng et al. [1995] to study time series of heart beat dynamics, since that time the DFA has been applied to a variety of topics studying long-range correlation in noisy, non-stationary monofractals e.g. behavioral ecological studies [MacIntosh, 2014], rapid eye movement [Shelhamer, 2005], financial market dynamics [Qiu et al., 2009] atomic vibrations in protein backbones [Morariu and Coza, 2003], and motion along tectonic plate boundaries [Tserolas et al., 2013]. The value of this method lies in the detection of long-range correlation in signals embedded in periodicities or external trends that may otherwise mask the true correlation properties of the signal or create false correlation. An alternate method to the MF-DFA is the wavelet transform modulus maxima (WTMM) method [Mallat and Hwang, 1992], described and applied to a variety of fields [Muzy et al., 1994; Arneodo et al., 1995]. The method has comparable capability in detecting multifractality in non-stationary, noisy series however the MF-DFA method provides the advantage of similar results with the ease of a less programming intensive application [Kantelhardt et al., 2002].

3.1 MF-DFA formalism

Consider a time series $x(k)$ of length $N$. Following Kantelhardt et al. [2002], we can calculate the accumulated times series or “profile” as

$$Y(i) = \sum_{k=1}^{i} [x(k) - \langle x \rangle]$$  \hspace{1cm} (3.1)

for $i = 1, ..., N$.

The profile is then divided into $N_s = \text{int}(N/s)$ non-overlapping segments of length (scale) $s$. Typically $N$ is not an exact multiple of $s$ so to account for missing pieces at the end of the series, the procedure is repeated by diving the profile starting at the end of the series. In total there are $2N_s$ segments at each scale.

The next step consists on calculating least squares fit of a polynomial $y_v$ to each segment $v$. The polynomial fit is then subtracted to each segment of the profile and the variance calculated by

$$F^2(v, s) = \frac{1}{s} \sum_{i=1}^{s} (Y[(v - 1)s + i] - y_v(i))^2$$  \hspace{1cm} (3.2)

for $v = 1, ..., N_s$. 

Starting from the end of the series

\[ F^2(v, s) \equiv \frac{1}{s} \sum_{i=1}^{s} (Y \left[ N - (v - N_s) s + 1 \right] - y_v(i))^2 \]  \hspace{1cm} (3.3)

for \( v = N_s + 1, ... , 2N_s \).

For the \( q \)th order fluctuation function, we average over all segments

\[ F_q(s) = \left( \frac{1}{2N_s} \sum_{v=1}^{2N_s} \left[ F^2(v, s) \right]^{q/2} \right)^{1/q} \]  \hspace{1cm} (3.4)

for \( q \neq 0 \).

For \( q = 0 \), we use the expression

\[ F_0(s) = \exp \left( \frac{1}{4N_s} \sum_{v=1}^{2N_s} \ln \left[ F^2(v, s) \right] \right) \]  \hspace{1cm} (3.5)

This process is repeated at various increasing scales, \( s \). It is recommended from previous analysis that \( s \) should not exceed \( N/4 \) for more reliable results [Kantelhardt et al., 2001]. If the fluctuation function, \( f_f \), scales as a power law, the signal is long-term correlated and the scaling exponent is identified as the generalized Hurst exponent, \( h(q) \).

\[ F_q(s) \sim s^{h(q)} \]  \hspace{1cm} (3.6)

Furthermore, the value of \( h(2) \), is used to determine the persistence of the signal categorized by its relationship to the Hurst exponent, \( H \): for \( h(2) < 1 \), the Hurst exponent is given by \( H = h(2) \) and for \( h(2) > 1 \) by \( H = h(2) - 1 \).

We calculate the scaling exponents at each \( q \) using a linear fit of \( F_q \) vs \( s \) on a log-log plot. To identify multifractality, we plot the scaling exponents, or \( h(q) \), versus \( q \). For a theoretical multifractal \( h(q) \) is a non-increasing function of \( q \) which tends asymptotically to constant values. The degree of multifractality can be quantified by \( \Delta h = h(q_{\text{max}}) - h(q_{\text{min}}) \). Conversely, in a monofractal signal, the scaling exponents are expected to be constant and independent of moment, \( q \), being fully described by one scaling exponent, \( H \), along with a zero degree of multifractality.

### 3.2 Spurious Multifractality

Previous analysis of the DFA method show that spurious multifractality may arise in correlated monofractal signals due to the multifractal bias of the Finite Size Effect (FSE) [Grech and Pamula, 2013]. From their analysis, it is recommended that the data size be a minimum of \( 2^{10} \) pixels in temporal length for more reliable estimates in \( h(q) \) [Grech
As will be shown in the next section, the observed signals show anti-correlated behavior, so we cannot directly apply the analytical correction of Grech and Pamula [2013]. Instead we estimate the FSE empirically by knowing that any multifractality detected in a monofractal is unauthentic and purely due to artifacts of the method and data size. To estimate the FSE we apply the MF-DFA to 2000 realizations of white noise time series of length 1,350 pixels as the observations. We find a spurious multifractality of degree $\Delta h = 0.16$. By subtracting this value from the original $\Delta h$ we define a "corrected" degree of multifractality.

Figure 3.1: Top row: The log-log plot of the fluctuation functions, $F_q(s)$ vs $s$ for selected $q$ values shifted vertically for clarity. Bottom row: Generalized Hurst exponents. Monofractal (left column) and Multifractal (right column)

Figure 3.1 displays the $f_f$s (top row) with associated $h(q)$ values (bottom row) for the monofractal, $H = 0.6$, (left) and multifractal (right) analyzed in the PDF section. The monofractal $f_f$s are visually parallel suggesting a constant $h(q)$ value. From the generalized exponents, the monofractal has an $H = h(2) - 1 = 0.59$ close to the expected $H = 0.60$ and original $\Delta h = 0.06$ showing a slight $q$ dependence. Given an FSE $\Delta h = 0.16$, we see that 100% of spread can be attributed to spurious multifractality. For the multifractal, $H = h(2) = 0.5$ and the original $\Delta h = 2.76$ and corrected $\Delta h = 2.60$ is still very high.

3.3 Noise Contamination

Additionally, we studied the effect of Gaussian noise (monofractal, $H = 0.5$) on the $h(q)$ spectrum in a common multifractal signal produced via multiplicative random cascade
The series begins with the zeroth iteration, \( n = 0 \) of a dataset \( x_i \) where \( i \) is the length of the series. At the starting point \( x_1^{n=0} = 1 \) and hereon each iteration doubles the length of the series such that \( x_{2l-1}^n = x_{2l-1}^{n-1}m_{2l-1} \) and \( x_{2l}^n = x_{2l}^{n-1}m_{2l} \), where \( l \) is an element in the series at iteration \( n \) and \( m \) is a normally distributed random number.

Following the exercise by Ludescher et al. [2011], we study the progression in the spread of \( h(q) \) values while adding noise at varying strengths: \( A = 0.01, 0.1, 1 \) and 10. At amplitude, \( A = 0.01 \), the spread of \( \Delta h \) is reduced compared to the pure multifractal while the overall shape remains unaffected. The reduction of multifractality comes mainly from the negative moments being shifted downward; this can be attributed to the noise affecting small events emphasized in negative moments. For \( A = 0.1, A = 1 \) and \( A = 10 \), the noise dominates the signal, reducing \( \Delta h \) even further as well as destroying the multifractal curve. Schematically, the effect of increasing noise causes the negative \( q \)'s to be shifted down while the positive \( q \)'s are shifted up, resulting in an underestimated degree of multifractality with a modified shape that tends to settle to a constant, \( h(q) = H \). Figure 3.2 visually demonstrates the progression of the diminishing multifractality versus increasing noise strength from left to right. This exercise was performed to show that multifractality can be reduced and the shape of the curve can be altered due to a noisy signal.

Figure 3.2: Generalized Hurst exponents of multifractal with added noise. FSE (\( \Delta h = 0.16 \)). Left to right: Noise \( A = 0.01 \): \( \Delta h = 1.25 \) actual \( \Delta h = 1.09 \), \( A = 0.1 \): \( \Delta h = 0.60 \) actual \( \Delta h = 0.44 \), \( A = 1 \): \( \Delta h = 0.34 \) actual \( \Delta h = 0.18 \), \( A = 10 \); \( \Delta h = 0.15 \) actual \( \Delta h = 0.00 \)

### 3.4 Sources of Multifractality

Multifractality can arise from two possible sources. It can occur through large fluctuations in a time series with a distribution characterized by fat tails, e.g. a log-normal or Levy function. A second way is through long-range correlations happening at small or large scales with values that are normally distributed e.g. Gaussian distribution. Once a signal is recognized to be multifractal, one can test the source by simply shuffling the values of original series. If long-term correlated, all memory in the series is lost through rearrangement of the values which destroys the original multifractality. However, if the multifractality still remains and is only reduced, it can be attributed to both sources [Kantelhardt et al., 2002].
4.1 MF-DFA analysis

We compute the fluctuation functions using previously developed C code by Goldberger et al. [2000] that we modified to include the range of moments desired. Our results were verified with our MATLAB code following the approach developed by Ihlen [2012]. The MF-DFA was performed for the core and peripheral weak emission regions as well as for the two loops. Every spatial pixel selected is associated with a distinct time series and the $f_f$s for a particular moment $q$ are averaged resulting in a single $F_q$ for each zone. The time series of 1,350 temporal pixels or 270 minutes were analyzed up to a maximum temporal scale $s = 64.6$ min, below the $N/4$ or 67.5 min maximum threshold. In the MF-DFA, the degree $m$ of the polynomial trend subtracted from the profile $Y(i)$ for a given data segment $s$, corresponds to subtracting a polynomial of degree $m-1$ from the original data $x(k)$. We investigated the results of using $m = 1, 2, 3$ and estimated that $m = 2$, a linear fit to the original series, was appropriate. We explore the range $-20 < q < 20$ at increments of 0.5 resulting in 81 $f_f$s.

As was stated before, a linear $f_f$ on a log-log plot indicates power law dependence. The scaling exponent $h(q)$ was identified as the slope resulting from the Chi-Square linear fit. To select the scaling ranges we look at all $F_q(s)$ curves and select the scales $s$ over which all curves present reasonable scaling. We avoid values less than $s \approx 10$ pixels or 3.2 min and $s > N/4$, or 67.5 min, since in those temporal scales the $f_f$s deviate the most from the true scaling values [Kantelhardt et al., 2001].

Before calculating the $f_f$ for general $q$ it is instructive to investigate $F_2$. In the case of a stationary time series or for fBm there is a relation between the spectral exponent and the generalized Hurst exponent: $\beta = 2h(2) - 1$ [Mandelbrot and Van ness, 1968; Heneghan and McDarby, 2000]. While there is no such relation for a multifractal it is useful to consider the information coming from the power spectra before selecting the scaling ranges. The power spectra for the 171Å signals in the core (not shown here), present a slight bulge in the 2.5-14 min range compatible with the excess emission observed by Ireland et al. [2014] in the 1-10mHz frequency range. This contribution is more noticeable for the emission at the footpoints and moss. The authors suggest that the excess emission is due to an oscillatory energy source lower in the atmosphere whose signal is able to reach the corona, as described by De Pontieu et al. [2004]. The weak emission region power spectra for the 171Å waveband does not present any excess emission. This is compatible with the fact that most of the pixels in this zone satisfy the positive lag condition associated with coronal emission.

Similarly the 335Å waveband emission at a hotter coronal temperature is not expected to have excess power in the acoustic frequency range. Based on these results we identify scaling ranges excluding the excess emission. For large temporal scales the lower end of the scaling range is taken at 14.8 min. Figure 4.1 shows an example of the linear fit to the average $F_2$ in the core for the 171Å and 335Å wavebands. The values of slopes
are summarized in Table 1. We note that the degree of anti-correlation for pixels with zero lag \( h(2) = 1.14 \pm 0.02 \) is stronger than for those with positive correlation \( h(2) = 1.26 \pm 0.03 \). In the case of the 335Å waveband for positive lags we also include a fit to the small scales since in this case there is no effect of excess emission from lower in the atmosphere. As we will show in the context of the model the decrease in the slope at small scales is due to the noise contribution in the signal.

![Figure 4.1: The average fluctuation function, \( F_2 \), with linear fit for the small scales (blue) or large scales (red) at the core for the 171Å positive lag values (left), 171Å zero lag values (middle) and 335Å positive lag value (left)](image)

To expand on how the noise is detected by the MF-DFA method I give a preview of some of the analysis results for the weak emission region in the 171Å waveband. The full details are presented later. Figure 4.2 displays the \( h(q) \) for positive lags in the 335-171 (left) and 211-171 (right) pairs. The generalized Hurst exponent for large scales with a range 14.8–41.8 min (black) displays the characteristic curve for a multifractal. For small scales for the fit 3.8 – 6.8 min (blue) the curve takes the shape of a deformed multifractal. The \( h(q) \) resembles the cases of a generated multifractal with added noise of amplitudes A=0.1 and A=1 as shown in Figure 3.2. While the larger scales still contain a noise contribution it is weaker than the signal. Based on these results we focus the analysis on the scaling of the large temporal scales which are least affected by the noise contributions.

4.2 MF-DFA: Core

For the 171Å emission in the core we find 775 (39%) positive lag pixels and 461 (23%) zero lag pixels. In the 335Å waveband only consider the positive pixels 960 (40%). The average fluctuation functions for selected values of \( q \) are shown in Figure 4.3 (top row) along with the associated values of \( h(q) \) calculated at the long range scales (bottom row). A summary of the numerical results is presented in Table 4.1.

At long ranges, the \( h(2) \) values for the positive lag pixels (coronal emission) indicate that the series are anti-persistent in both spectral lines where \( H = h(2) - 1 = 0.26 \pm 0.03, 0.29\pm0.03 \) for 171Å and 335Å respectively. The zero lag pixels in the 171Å emission show a stronger anti-persistence with smaller value of the Hurst exponent: \( H = h(2) - 1 = 0.14 \pm 0.02 \). The degree of multifractality is smaller for the 335Å emission as compared to
Figure 4.2: Generalized Hurst exponent, $h(q)$, in the weak emission zone of the 171Å band. Large scales for temporal range 14.8-41.8 min. are in black and small scales for the temporal range 3.8-6.8 min are in blue. For positive temporal offset values in (left) lag pair 335-171Å and (right) lag in pair 211-171Å.

Figure 4.3: Top row: The log-log plot of the average fluctuation functions, $F_q(s)$ vs s. We plot every 5th function shifted vertically for clarity. Bottom row: Generalized Hurst exponent with associated error bars for the Core region. Left column: Positive lag values in the 171Å, Center column: Zero lag values in the 171Å, and Right column: Positive lag values in the 335Å.
Figure 4.4: $F_q(s)$ vs $s$ (right), showing every 5th function shifted vertically for clarity, and generalized Hurst exponent (left) for the positive and zero lag values in core 171Å region when the time series is randomly shuffled. The value of $h(2) = 0.50$ corresponds to an uncorrelated signal. Given the estimated FSE, $\Delta h = 0.16$, the multifractal effect is 100% due to spurious multifractality.

the one in the 171Å because of the relative stronger noise contribution. After accounting for spurious effects, the multifractality remains significant in all channels. 64.4 (171Å) and 70% (335Å) of the original $\Delta h$ value can be attributed to multifractality for the positive lag values while 66.7% (171Å) for the zero lag values.

Figure 4.4 shows the average $F_q$ and corresponding $h(q)$ for the shuffled time series corresponding to positive lag pixels with emission in the 171Å waveband. The degree of multifractality is greatly reduced such after accounting for FSE ($\Delta h = 0.16$), $\Delta h$ is effectively zero. Thus we conclude that the multifractality arising in the EUV emission is caused by the long-range memory in the time series and not due to the heavy tails in the log-normal intensity distributions.

4.3 MF-DFA: Weak Emission

The same lag criteria was used at the weak emission zone however the filtering was not performed in this region as described in Section 3.1.2. In this area, the 171Å & 335Å resulted in 1048 pixels (48%) and 873 pixels (40%) positive values of the total unfiltered pixels coinciding between lag pairs. However, there were very few zero lag pixels (29 (1%)). Given these values, we present only the positive lag results for the weak emission zone.

The MF-DFA results are displayed in Figure 4.5 showing the $ffs$ (top row) along with the corresponding $h(q)$ spectrum (bottom row). The positive lag values in the 171Å maintain a multifractal shape while the 335Å is significantly contaminated with noise causing it to flatten out. The $h(q)$s of the positive lag values in the 335Å settle to almost a constant value where $h(q) = 0.79 \pm 0.01$. This value of the Hurst exponents indicates a persistent series, with essentially zero multifractality, $\Delta h = 0 \pm 0.02$, after accounting for FSE. The noise contamination to the 335Å overwhelms the signal. The results is similar to the
multifractal with noise amplitude $A=10$ previously shown where $H = h(2) = 0.5$ corresponding to the uncorrelated white noise. For the 171Å, the signal remains anti-persistent, $H = h(2) - 1 = 0.38 \pm 0.01$, albeit with a lower degree of anti-correlation than the core. 67.3% of the original $\Delta h$ value can be attributed to multifractal properties.

Figure 4.5: Top row: The log-log plot of the average fluctuation functions, $F_q(s)$ vs $s$. We plot every 5th function shifted vertically for clarity. Bottom row: Generalized Hurst exponent with associated error bars for the Weak Emission region. Left column: Positive lag values in the 171Å, Right column: Positive lag values in the 335Å

4.4 MF-DFA: Loops

The two loops presented in this paper were previously mapped in Cadavid et al. [2014] using images in the 94Å waveband. The loop structure included in the MF-DFA analysis uses 4 pixels above and below the loop positions. Just as in the core, the loop pixels where filtered and the dual lag image criteria placed on the loops results in 233 pixels (71%) in Loop 1 and 242 (41%) in Loop 2 of the total unfiltered pixels with positive lag values. However, only Loop 2 had a significant number of overlapping zero lag values to report at 99 points (17%) compared to < 0.1% for Loop 1. Figure 4.6, top row, shows an average of $F_2$ with corresponding linear fit showing anti-persistent correlation for Loop 1 (positive) and Loop 2 (positive and zero) in 171Å band. The positive lag values in Loop 1 and 2 show compatible correlation to those seen in the core/weak emission positive lag values. Similarly, we find agreement in $h(2)$ between the zero lag pixels in Loop 2 to zero lag pixels analyzed at the core of the 171Å. As displayed in the $h(q)$s of Figure 4.6, bottom row,
multifractality is present in both Loop 1 (positive) and 2 (positive and negative) with 72.0%, 71.9% & 76.1%, respectively, of the original $\Delta h$ that can be attributed to multifractality after accounting for the FSE.

Figure 4.6: Top row: The log-log plot of the average fluctuation function, $F_q$ vs. $s$. Bottom row: Generalized Hurst exponent with associated error bars all in the 171Å core region. Left column: Loop 1 positive lag values, Center column: Loop 2 positive lag values and Right column: Loop 2 zero lag values.
Table 4.1: Summary of the MF-DFA results for the core, weak emission and loops. Along with the average Lag of positive pixels in each lag pair. Total percentage of points use in the MF-DFA analysis at the particular waveband (171Å and 335Å). The fit range of the fluctuation functions. As well as the $h(2)$, $\Delta h$ and corrected $\Delta h$.

<table>
<thead>
<tr>
<th>Region</th>
<th>Avg. Lag 335-171 (min)</th>
<th>Avg. Lag 211-171 (min)</th>
<th>% of points</th>
<th>Wavelength (Å)</th>
<th>Fit Range (min)</th>
<th>$h(2)$</th>
<th>Original $\Delta h$</th>
<th>Corrected $\Delta h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>38.0</td>
<td>29.4</td>
<td>39</td>
<td>171</td>
<td>14.8 – 32.2</td>
<td>1.26 ± 0.03</td>
<td>0.45 ± 0.05</td>
<td>0.29 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>23</td>
<td>171</td>
<td>14.8 – 41.8</td>
<td>1.14 ± 0.02</td>
<td>0.48 ± 0.03</td>
<td>0.32 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>39.9</td>
<td>29.1</td>
<td>40</td>
<td>335</td>
<td>16.2 – 41.8</td>
<td>1.29 ± 0.03</td>
<td>0.20 ± 0.06</td>
<td>0.14 ± 0.04</td>
</tr>
<tr>
<td>Weak</td>
<td>46.8</td>
<td>35.2</td>
<td>48</td>
<td>171</td>
<td>14.8 – 41.8</td>
<td>1.38 ± 0.01</td>
<td>0.49 ± 0.02</td>
<td>0.33 ± 0.02</td>
</tr>
<tr>
<td>Emission</td>
<td>39.9</td>
<td>29.1</td>
<td>40</td>
<td>335</td>
<td>16.2 – 41.8</td>
<td>0.79 ± 0.01</td>
<td>0.05 ± 0.02</td>
<td>0.00 ± 0.02</td>
</tr>
<tr>
<td>Loop 1</td>
<td>48.9</td>
<td>17.2</td>
<td>71</td>
<td>171</td>
<td>14.8 – 35.2</td>
<td>1.41 ± 0.03</td>
<td>0.50 ± 0.05</td>
<td>0.36 ± 0.03</td>
</tr>
<tr>
<td>Loop 2</td>
<td>33.7</td>
<td>25.8</td>
<td>41</td>
<td>171</td>
<td>14.8 – 41.8</td>
<td>1.34 ± 0.04</td>
<td>0.57 ± 0.06</td>
<td>0.41 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>17</td>
<td>171</td>
<td>14.8 – 41.8</td>
<td>1.15 ± 0.03</td>
<td>0.67 ± 0.03</td>
<td>0.51 ± 0.03</td>
</tr>
</tbody>
</table>
Chapter 5
PDFs and MF-DFA analysis in the models

5.1 Synthetic light curves

In order to interpret the scaling properties exhibited in the data, we utilize a simple ‘phenomelogical’ model developed by Pauluhn and Solanki [2006]. The model statistically produces light curves constrained to a log-normal distribution of intensities and energies displaying a power law distribution representative of EUV radiances of the quiet sun and lower corona. The model assumes all emission is due to nanoflares. We select the parameters to the simulation emission to match scaling results observed at different AIA wavebands to gain some insight.

The method is based on impulsive nanoflare heating modeled through a series of overlapping of random “bursts”. The impulses are given a random probability of occurring, $P_f$, at each timestep and magnitude, $I_0$, chosen from a range of values that satisfy a power law distribution with exponent, $\delta$. Correlation in the modeled data is developed by the rise, $\tau_r$, and a longer decay time, $\tau_d$, parameters given to the impulses. Thus, for example, we apply to the data points lying within $k$ steps after the flare point an added intensity boost of $I_0e^{(-k/\tau_d)}$ where $k < \tau_d$. And similarly for leading times and $\tau_r$. We select parameters to produce light curves with comparable multifractal properties of observations. The amplitude, $I_0$, is in the range 0.01-0.1 sampled from a power law distribution with exponent $\delta = 2.1$. The impulses have an exponential rising time $\tau_r = 1.5$ min, an exponential decay time $\tau_d = 6$ min, and the probability of a “burst” at any given step is $P_f = 0.1$. For analysis, we generate a set of 2000 realizations of light curves of 1500 pixels of temporal length, using pixels from 250-1500, 1350 pixels total, to resemble the data dimensions. We chose to discard the first 250 pixels during the initial ramp up so the model would be analyzed in a statistically steady state.

Figure 5.2 displays a sample of normalized light curves from the solar observations: the core (A) and weak emission (B), to compare with the generated light curves: pure model with no noise (C) and model with added noise (D) produced with selected model parameters. The light curves in all four plots are an average of 20-30 time series. Figure 5.1 show the location of the light curves of the core and weak emission, denoted by the red pixels, chosen at a random location in the box. Time series plotted in (C) and (D) are an average of synthetic light curves chosen from an arbitrary location among the 2000 realizations. A small amplitude of noise was added to the light curves in (D) to resemble the variability seen at the core.

Figure 5.3, top row, shows that generalized Hurst exponent and, bottom row, the corresponding PDF of increments for the pure model (right column) and model with added noise (left column). The multifractality in the Generalized Hurst exponent appear in both due to the correlation that forms from the exponential rise and decay of the bursts. Both have similar $h(2)$ values indicating anti-persistence. The pure model shows a higher multifractality than that of the light curves with added noise, $\Delta h = 0.66 \pm 0.01$ versus $\Delta h = 0.56 \pm 0.01$, after accounting for FSE. The presence of noise underestimates the multifractality from the
Figure 5.1: A temporal average of AR11250 from 8 evenly separated frames within the observation time frame in the 171Å band. The pixels denoted in red are used as an illustration of the light curves observed in the core and weak emission region for Figure 5.2.

Figure 5.2: Example time series from observations versus the P&S model. For (A) and (B), the light curve is an average of 36 surrounding time series in the given location, see Figure 5.1. Top to bottom: (A) Average time series from pixels in the core (B) Average time series from pixels at the weak emission zone. (C) Model for impulsive heating generated by the S&P model (D) Model for impulsive heating with added noise.
Figure 5.3: Generalized Hurst exponent for Pure P&S (Top, left) and associated probability distribution function of increments (bottom, left). Generalized Hurst exponents for P&S model with added Gaussian noise (Top, right) and associated probability distribution function of increments (bottom, right). The scaling range for the $F_q(s)$ vs $s$ fit is 18-42min. Top left: At the large scales 29.6-64.6min, $h(2) = 1.38 \pm 0.01$, $\Delta h = 0.82 \pm 0.01$. Top right: $h(2) = 1.37 \pm 0.01$, $\Delta h = 0.72 \pm 0.01$. Bottom row: PDF scales: 24 s, 1.6 min, 12.8 min, 1.7 hr.
original signal but is not dominant in the process. The $h(q)$ values fall within the observational spectrum range, $0.5 < h(q) < 2$.

In addition, we find the probability distributions for the pure and noisy time series to be statistically incompatible to distributions from observations. In the observations, skewness remains close to zero for both wavebands, while the synthetic light curves are highly asymmetric with skewness $> 1$ in all cases. The highest skewness appears at the smallest scales and decreases at higher increment values. In Figure 5.3, bottom left, the skewness values beginning at the small scales from top to bottom, skewness=2.2, 1.9, 1.6, 1.4, 1.5 (pure model), and, bottom right, skewness=1.1, 1.5, 1.5, 1.5, 1.4 (noisy model). The skewness for the pure model shows a peaked kurtosis at small scales that decreases close to zero at the larger scales: kurtosis=6.9, 4.72, 3.13, 1.66, 0.21. The model with added noise shows much lower, nearly constant kurtosis at all scales, kurtosis=0.96, 0.97, 0.75, 0.90, 0.55.

Furthermore, given a separate data set generated with similar model parameters and dimensions, we superimpose uncorrelated Gaussian noise to the average fluctuation function, $F_2$, to study the effects to the scaling properties and correlation. Hu et al. [2001] shows the fluctuation function of the sum of two uncorrelated signals, $x(t)$ and $y(t)$ given $F_2^x(s)$ and $F_2^y(s)$, can be expressed as

$$F_2^{x+y}(s) = \sqrt{(F_2^x(s))^2 + (F_2^y(s))^2} \quad (5.1)$$

Given this relationship, due to the noise fluctuations affecting the statistics of the small scales, we find that by adding a small amplitude of noise the scaling values gradually move closer to the uncorrelated value of the noise ($H=0.5$). Figure 5.4, displays the average $F_2$ for the pure model with no noise (left) and the model with noise amplitude 0.05 (right) with corresponding slope at the small and large scales. In the pure model, the slope at the small scales (1.4-6.8 min) is very steep not compatible with data results and outside of the traditional Hurst exponent range. The large scales (17.6-45.6 min) have an $h(2)$ indicating anti-correlation in the signal close to observational results at similar temporal regime. In the case with added noise at amplitude 0.05, the small scale slope decreases to a correlated value $h(2) = 0.58 \pm 0.01$. Given the noise amplitude which can be up to 50% of intensity amplitude given to bursts of the light curves, the small scales are dominated by the noise. At the large scales the slope indicates stronger anti-correlation than in the case with no noise, $H = h(2) - 1 = 0.12 \pm 0.01$. As expected, the corresponding multifractality is reduced to $\Delta h = 0.30 \pm 0.02$, from the value of the pure model $\Delta h = 0.58 \pm 0.04$, due to the noise. Given this noise amplitude, the scaling at the small scales of the model is compatible to positive lag values at the core in the 335Å waveband. In addition, due to the added noise, the PDFs of increments (not shown here) are Gaussian at all scales.

Although we did not conduct a thorough investigation of model parameters, the primary use of the model was to investigate if the model with added noise can match the correlation, degree of multifractality and PDF statistics seen in the observations results. We see that a model of impulsive bursts characterized by amplitudes following a power law distribution
can produce the degree of correlation in solar observations. The superposition of noise provides an $h(2)$ and degree of multifractality comparable to those encountered in the AIA signals with varying SNR. On the other hand, the model was not able to reproduce the statistics found in the distributions of increments. In the pure model, the distribution are asymmetric and highly skewed. When adding a small amplitude of noise, the distributions become Gaussian at all scales. As stated earlier, one criteria of the model is a lognormal distribution of intensities. The intensity distributions of the model for given parameter best fit the weak emission region of the 171Å band however even in this case the PDF increments do not match. Given these results, we see that the distribution of increments are important to the analysis and that multifractality on its own is not a sufficient constraint for models.

### 5.2 The reduced MHD model

This section analyses the energy release from a more realistic magnetically dominated MHD turbulent model. We analyzed ohmic dissipation outputs from a previous paper run independently of this analysis. The model studies the Parker field line tangling conjecture, where it is proposed that continuous magnetic footpoint displacement leads to tangential discontinuities in the form of current sheets responsible for heating the corona through turbulent cascades of energy [Rappazzo et al., 2008, 2013]. The energy output can be most related to the hot channels, 131Å & 94Å, in the EUV which carry the most information of energy dissipated however the noise in those channels inhibits any useful comparison from the MF-DFA analysis. Instead, we compare loop structures in the 171Å that contain the most pure form of the dynamic process.

The simulations study the dynamics of a rectangular coronal loop with strong axial magnetic field via reduced MHD framework. The simulation box contains uniform magnetic field lines embedded in plasma that are slowly shuffled by photospheric velocity.
fields guides at the boundaries, used to mimic convective granular motions. The magnetic Reynolds number, $Re_m = \frac{Lv}{\eta}$, where $L$ is the typical length scale, $v$ is the typical plasma velocity flow and $\eta = \frac{1}{\mu_0\sigma_0}$ is magnetic diffusivity defined by the permeability of free space, $\mu_0$, and the electrical conductivity of the material, $\sigma_0$, is defined as the ratio of the convective term to the diffusive term in the induction equation. Typically a high value ($Re \gg 1$) such as that for the corona, $Re_m \approx 10^8 - 10^{12}$, describes a magnetic field that is advected by the plasma flow [Aschwanden, 2006]. The Alfvén velocity, $v_A(r)$, is a function of local electron density, $n_e(r)$, and magnetic field, $B(r)$ at a particular height given by $v_A(r) = \frac{B(r)}{\sqrt{n_e(r)}}$. The simulation assumes homogeneity with constant density in the loop while varying the Alfvén velocity and, in turn, the magnetic field.

In terms in the simulation, in order to resolve an inertial range within the simulation resolution, a hyperdiffusive term, $n = 4$, must be used in the Laplacian. There is an initial linear stage for the ohmic dissipation where the loop velocity field is governed by the the photospheric field while the magnetic field in the loop grows linearly. The onset of non-linear dynamics occurs after this stage, where energy balance occurs between the Poynting flux and energy released in the form of ohmic and viscous dissipation at the small scales. The ohmic dissipation dominates over viscous dissipation identifying the magnetic reconnection events as the main source of energy release. From this point, the system does not return to a state of energy accumulation and the system remains in the inertial range until the end of the simulation.

For the cases with hyperdiffusion and high Reynolds number, a dependence of Alfvén velocity, or equivalently the axial magnetic field strength, $B_z$, is found in the Energy Spectra corresponding to weak turbulence at smaller scales and stronger turbulence at larger scales with increasing Alfvén velocity [Rappazzo et al., 2008]. Kolmogorov turbulence is famously given the scaling of -5/3 for hydrodynamics or -3/2 for the magnetohydrodynamic case. However, in an an-isotropic situation given by the energy cascade occurring only on the perpendicular plane to the axial magnetic field, $B_z$, such as this simulation the power law scales to -2 [Rappazzo et al., 2008; Ng and Bhattacharjee, 1997; Galtier et al., 2000].

The model parameters and analysis results to the ohmic dissipation outputs are described in Table 5.1. The model most comparable to solar observations results is model I given parameters: axial length, $L$, corresponding to 40,000 km loop length, Alfvén velocity $v_A = 1,000 \text{ km s}^{-1}$ corresponding to $c_A = v_A/v_{ph} = 1,000$, Reynolds number $Re_A = 10^{19}$, velocity fields applied to the top ($z = L$) and bottom ($z = 0$) of simulation box, and total simulation duration of 502 axial crossing times $\tau_A = L/v_A$ corresponding to 334.7 min. We excluded the initial linear transient, shortening the series to 315.4 min (4,717 pixels) in temporal length with corresponding timestep $0.012 \text{ sec pixel}^{-1}$. Figure 5.5, left, displays the average fluctuation function, $F_2$, showing the slope for the small and large scaling range. For scaling range 0.9-2.9 min $h(2) = 2.20 \pm 0.05$, the $h(2)$ value cannot be directly related to the Hurst exponent because it is out of the traditional range however still exhibits strong correlations. The large value seen at the small scales is comparable to the value found in the pure P&S model at similar temporal ranges (see Figure 5.4). For
the large scales, $h(2) = 1.07 \pm 0.04$ indicates an anti-persistent signal consistent with the results for the core and weak emission zones though much stronger anti-correlation. The generalized Hurst exponent for the large scales displayed in Figure 5.5 (center) exhibits multifractal structure with $\Delta h = 0.68 \pm 0.11$, after accounting for FSE, much larger than any solar results. The shuffled time series has value $h(2) = H = 0.43 \pm 0.02$ showing small anti-correlation while diminishing the multifractal spectrum to $\Delta h = 0.06 \pm 0.04$, close to zero however some multifractality remains. Figure 5.5 (right) shows the distribution of increments for scales 0.1, 0.5, 2.1, 8.6, 26 min with corresponding kurtosis =1.8, 1.7, 0.4, 0.01, 0.2. The values are very small and close to Gaussian, in all cases. We do not observe evidence of 'fat tails' consistent with no major energy accumulation in the simulation.

<table>
<thead>
<tr>
<th>Model</th>
<th>$V_A (km/s)$</th>
<th>Re</th>
<th>Fit Range</th>
<th>$h(2)$ small scale</th>
<th>$\Delta h$ small scale</th>
<th>Fit Range</th>
<th>$h(2)$ large scale</th>
<th>$\Delta h$ large scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>200</td>
<td>$8 \times 10^2$</td>
<td>2.3 – 13</td>
<td>2.46 ± 0.02</td>
<td>1.53 ± 0.24</td>
<td>87 – 415</td>
<td>0.62 ± 0.04</td>
<td>0.59 ± 0.10</td>
</tr>
<tr>
<td>F</td>
<td>50</td>
<td>$3 \times 10^{10}$</td>
<td>1.8 – 13</td>
<td>2.09 ± 0.01</td>
<td>0.34 ± 0.08</td>
<td>148 – 591</td>
<td>0.88 ± 0.06</td>
<td>0.63 ± 0.16</td>
</tr>
<tr>
<td>G</td>
<td>200</td>
<td>$10^{19}$</td>
<td>1.1 – 14</td>
<td>2.05 ± 0.01</td>
<td>0.22 ± 0.05</td>
<td>56 – 348</td>
<td>0.63 ± 0.03</td>
<td>0.57 ± 0.10</td>
</tr>
<tr>
<td>H</td>
<td>400</td>
<td>$10^{20}$</td>
<td>1.2 – 12</td>
<td>2.16 ± 0.04</td>
<td>0.23 ± 0.10</td>
<td>1.2 – 12</td>
<td>2.16 ± 0.04</td>
<td>0.23 ± 0.10</td>
</tr>
<tr>
<td>I</td>
<td>1,000</td>
<td>$10^{19}$</td>
<td>0.9 – 2.9</td>
<td>2.20 ± 0.04</td>
<td>0.58 ± 0.29</td>
<td>9.9 – 61</td>
<td>1.07 ± 0.04</td>
<td>0.80 ± 0.11</td>
</tr>
</tbody>
</table>

Table 5.1: Ohmic dissipation model characteristics from Rappazzo et al. [2008, 2013] along with MF-DFA results for small and large scaling regimes.

Figure 5.5: Left: Log-log plot of the average $F_2$, for the ohmic dissipation of the reduced MHD simulation. Center: Generalized Hurst exponent at the large scales. Right: PDF at scales (top to bottom): 0.1, 1.5, 2.1, 8.6, 26 min. The PDFs were shifted down for clarity.

In a 3D MHD model by Dahlburg et al. [2016] they model the dynamics coronal loop with similar features as the previous simulations using the HYPERION code. Again, the
magnetic field lines are anchored at the bottom of the box shuffled by random motions. Non-linear dynamics sets in through the formation and dissipation of current sheets where the heating occurs non-uniformly heating different plasma mass creating a multi-thermal loop. The density and temperature from simulations are used to create synthetic light curves to be used in the creation of the Differential Emission Measure (DEM) to compare to emission observed by the Extreme ultraviolet Imaging Spectrometer (EIS). They find compatibility in the in both peak temperature and emission distribution for the 171Å & 195Å filter bandpasses. Given the similarity in the approaches of the two MHD models, we find the comparison between intensity fluctuation results of the solar data in the 171Å band relevant to the ohmic dissipation fluctuation results observed in model I.

Due to the turbulence in the model, multifractality is expected to appear in the fluctuations of the ohmic dissipation. While the results of the models do not all quantitatively agree with observations, we do emphasize that at large scales the signal develops multifractality caused by long range correlation and the correlation is anti-persistent. The increment distributions indicate there is less variability in the model time series than in the AIA signals of the loops. A possible reason for this can be that the model does not have energy accumulation that results in no large energy releases.
In summary, the techniques discussed have been valuable for observing varying degrees of multifractality in the intensity fluctuations of the AR. To use the complementary techniques of PDF of increments and MF-DFA to uncover and quantify the multifractality in the EUV emission, it is necessary to use the cooler, higher intensity 171Å waveband because it is the least affected by the noise level. We found that the multifractality for temporal scales in the approximate range 15 to 45 min is due to long-range correlations since the multiscaling properties disappear for the shuffled time series. The original lag technique was extended to identify pixels in which both the 335-171 and 211-171 pairs had simultaneously positive or zero lags. The regions with a positive temporal offset were associated with coronal emission, while the zero lag pixels were associated with TR emission. Given this distinction, we found that for all the physical regions and for both positive and zero lag pixels, the 171Å emission is anti-persistent. For the zero-lag pixels, the $h(2)$s at the core and Loop 2, both display stronger anti-correlation compared to the positive temporal offsets. The 335Å waveband emission in the core also showed anti-persistence but a weak degree of multifractality. In the weak emission region where the noise dominates the multifractality disappears and the Hurst exponent of corresponds to that of a positively correlated monofractal. The noise contribution is also apparent in the quasi-Gaussian shape for PDFs of increments.

In the spirit of inquiry, we have examined the 171Å waveband emission above an active region near the limb which should contain no TR emission. Indeed the 335-171 lag pair shows very few zero lag pixels for a region just above the disk and no zero lag pixels if the region is far above the disk. In preliminary results, for locations far above the disk we find correlation similar to the line-of-sight results with $0 < H < 0.5$ and higher degrees of multifractality ~0.45. As we move to locations closer to the disk we see similar degrees of multifractality, however, both positive and zero lag pixels, show positive correlations $0.5 < H < 1$. Further analysis is needed to investigate off-the-limb results.

The model by Pauluhn and Solanki [2006] serves to demonstrate that an impulsive heating model, given an exponential rise and decay, with added noise produces light curves with anti-correlation compatible to solar emission however not compatible with increment PDF statistics. The reduced MHD simulations, although not parameterized to fit any observational conditions, produced multifractals as expected due to the turbulent energy cascade in the model. In the large scales, with the exception of model I, the $h(2)$ values were not compatible to solar results. In addition, the simulations did not match observational PDF statistics. The analysis of the models show that multifractality is not a sufficient constraint and the PDFs are needed to complement this analysis. Although these models did not match well with solar results, we envision using the MF-DFA in concert with the distribution of increments on future models optimized to specific observation conditions.
Bibliography


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