Random Effects Modeling on Electroencephalography

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the requirements for
the Degree

Master of Arts

In

Psychology: Mind, Brain, and Behavior

by

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Certification of Approval

I certify that I have read Random Effects Modeling on Electroencephalography by Alvaro Ernesto Ramos, and that in my opinion this work meets the criteria for approving a thesis submitted in partial fulfillment of the requirement for the degree Master of Arts in Psychology: Mind, Brain, and Behavior at San Francisco State University.

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Abstract

This study is a statistical analysis of repeated measures resting-state electroencephalography (EEG) data in a 2 (eyes-open and eyes-closed) x 3 (first 10 seconds, second 10 seconds, and third 10 seconds) design. Two analyses of the same data are conducted to determine the relative merits of each approach. One analysis is a repeated-measures analysis of variance examining EEG frequencies (Alpha, Beta, Delta, and Theta). The other analysis is a linear mixed effects regression examining the identical outcome and design. The main purpose of this thesis is to determine if linear mixed effects regression is a better statistical method than repeated-measures analysis of variance, specifically in terms of providing more precision on the effect estimates in terms of smaller error terms per effect (viz., standard error of the betas). For all EEG frequencies, a total of 20 standard errors out of 24 standard errors were smaller for the linear mixed effects regression models as compared to repeated-measures analyses of variance.
Acknowledgements

Thank you to Prof. Charlotte Tate for statistical expertise and Prof. Mark Geisler for providing EEG data that supported the bridge between Psychophysiology and Quantitative Psychology for the advancement of psychological science. I would like to express gratitude to my parents and family for supporting my pursuit of higher education.
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Introduction

My research interest is to analyze resting-state electroencephalography (EEG) oscillations in the structure of a repeated-measures research design using two different statistical techniques to advance the field of EEG data analysis. EEG is measurable as amplitudes (i.e. standard deviation of a segment of electrical activity) of the oscillations. The resting-state is a constant condition without stimuli or behavioral events (Snyder & Raichle, 2012, p. 902).

Traditionally, univariate analysis of variance (ANOVA)--including single-factor repeated measures ANOVA (RMANOVA)--or multivariate analysis of variance (MANOVA) are the most common types of statistical analyses used on repeated measures designs for studies in cognition, neurology, and psychophysiology (Baayen et al., 2008 p. 409; Dien & Santuzzi, 2004).

Statistical Elements within Repeated Measures Analysis of Variance

A repeated-measures design of two conditions can capture individual differences among participants in their individual response profiles to any task which can be estimated across the two conditions. (Of note, this meaning of “individual differences” is specific to the task responses of participants [not personality traits]). Repeated-measures ANOVA expresses treatment effects in terms of ANOVA and uses standard $F$ tests to assess their significance (Bagiella et al., 2000, p. 14). Mathematically speaking, the sum of squared persons and sum of squared errors are partitioned from the sum of squared within (Field et al., 2012, p. 555).

Traditionally, the sum of squared persons is subtracted out of the sums of squared within term (Field et al., 2012, p. 556), which means that information about individual response profiles is effectively “lost” in the partitioning of variance. When response profiles are subtracted out of the
larger within-subjects error term, the calculated sum of squared errors (i.e., all error minus the response profiles) is the chance variation estimate that becomes the denominator of the observed $F$ ratio (e.g., Field et al., 2012, p. 560-561). In effect, all unmeasured sources of variability except the response profiles are now the effective chance variation for RMANOVA. However, the response profiles are not used in another part of the RMANOVA equation (e.g., such as part of an intercept effect that could itself partition variance). Consequently, statistical analysis for EEG is not currently using all the measured information researchers have to be assigned to an estimated effect term because they are losing each person’s response profile (by subtracting only, to create, on average, a smaller chance variation error term for any observed $F$-ratio). A review of the literature on the statistical methods for repeated-measures data in psychophysiological research reveals potential alternatives to traditional statistical analyses, such as the application of mixed effects modeling (e.g., Baayen et al., 2008, p. 409; Bagiella et al., 2000). Bates et al. (2015) describe the “term ‘mixed’ or, more fully, ‘mixed effects’, denotes a model that incorporates both fixed- and random-effects terms in a linear predictor expression from which the conditional mean of the response can be evaluated” (p. 1). Mixed effects modeling is a multivariate statistical technique in which variables from the data construct a regression model. Specifically, certain components of the model are assumed fixed effects (roughly, an unweighted average across the responses), while a component that (e.g., the y-axis or outcome intercept) varies across a grouping variable is called a random effect (Field et al., 2012, p. 862-865). Mixed effects models can have fixed and random effects on variables of interest and are flexible to the design of a study and the goals of an analyst. For example, a fixed effect term can be the conditions or the time of a sample. When the conditions are modeled as fixed effects, all
possible treatment conditions that a researcher is interested in are present in the experiment (Field et al., 2012, p. 862). Fixed effects are treated “normally” in ANOVA or regression as an unweighted average across the responses. They are interpreted as the non-varying element of the design that should replicate under similar conditions in another study. Whereas a random effect term could be on human participants or stimuli. When human participants are modeled as random effects, the experiment contains only a random (sub)sample of possible treatment conditions (Field et al., 2012, p. 862). Random effects as such (as random subsets of possible treatments or elements) are never treated in traditional ANOVA or regression because these analyses are fixed-effects only models. They are interpreted as a new varying element of the design that is not expected to replicate under similar conditions in another study. An example of the application of a random effect in a psychophysiological experiment is when a participant makes a fist with their hand for one minute followed by relaxation of their hand for one minute (2 levels [i.e., fist, relaxed] of single variable measured, both of which are experienced by the participant [viz., repeated measures]. Participants can be a random intercept completing the 2 conditions. Individual performance has measurable effects on the amplitudes of the electrocardiogram (ECG) recording of pulse in each participant’s arm. In a linear mixed model (LMM), the sum of squared effect for persons (aka response profiles on the task) can be assigned to the random intercept (Bates et al., 2015). In this manner, the sum of squared persons is effectively used somewhere in the larger equation as an effect term in-itself to partition variance on the measured outcome itself (e.g., amplitudes), instead of removing it from chance variation calculations only. In this case, the random intercept would weight all the responses on the outcome (e.g., an EEG amplitude) by each individual respondent’s response profile to develop a
‘line of best fit.’ Overall, the LMM illustrates that, “each case has its own regression equation when random slopes and intercepts are specified, it is possible to evaluate whether individuals do indeed differ in their mean response and/or in their pattern of responses over the repeated measure” (Tabachnick & Fidell, 2018, p. 818). It is possible to convert repeated-measures ANOVA to a repeated-measures regression, by performing an ordinary least squares (OLS) regression on the ANOVA (Pedhazur, 1982; Tabachnick & Fidell, 2013, pp. 156-158). In OLS we create one ‘line of best fit’ and average in an unweighted average (across all persons and their response profiles). A knowable mathematical consequence of using response profiles as a random intercept is that assigning persons variability to the intercept term will, on average, further reduce the standard error terms for all effects. (This is because assigning effects as “numerator” terms requires calculating suitable chance variation for that numerator term, and all chance variation comes from the larger within-subject error term.) Using unweighted averages is imprecise because some information is lost (e.g., at the person-level). Moreover, at the broadest interpretive level (as described above), treating participants as a random effect suggests that the specific participants in the study are simply a random subsample of all possible participants. In order to forge precision, we could create regression equations for each person and average them together, treating persons as a random intercept on the amplitudes of interest.

Furthermore, there are advantages to using mixed effects models instead of traditional statistical analyses for EEG data analysis, including pre-processed EEG data (Bagiella et al., 2000). Mixed models use all the available data and incorporate error variance that permit correct and efficient estimation of treatment effects under a broad range of variance structures (Bagiella et al., 2000, p. 18). Repeated-measures ANOVA yields correct tests only under conditions of
sphericity, otherwise a sphericity correction is necessary (Bagiella et al., 2000, p. 18). Journal policy in psychophysiological research addresses problems from repeated-measures experimental designs by adopting statistical procedures that consider the correlated data (Baayen et al., 2008, p. 409; Jennings, 1987; Vasey & Thayer, 1987). For example, violations of sphericity require a correction such as the Greenhouse-Geisser (Greenhouse & Geisser, 1959) or the Huynh-Feldt (Huynh & Feldt, 1976). MANOVA admits a very general variance model, and, consequently, gives correct tests and estimates under very broad assumptions (Bagiella et al., 2000, p. 18-19). Nonetheless, the MANOVA procedure is focused on canonical variates (viz., linear combinations of variables) as the main analysis object, only allowing for univariate tests as follow-ups or peel-offs, and often, via conventional practices, without appropriate Type I error control (cf. Tabachnik & Fidell, 2018, chap. 7, p. 245-313). Finally, although mixed models are more computationally demanding than traditional methods (Bagiella et al., 2000, p. 19), thankfully advancements in the computational sciences have led to plenty of computational power in the twenty-first century.

**Overview of the Study**

My topic of interest is whether LMM analysis of pre-processed EEG data can provide more precision on the estimated effects than RMANOVA of the same dataset. The standard for precision that we use is the size of the standard errors per term in the analysis model. As we developed above, smaller standard errors are a reasonable indicator of increased precision (because the error due to chance is being reduced by how the algebra has been developed). In terms of EEG methods, we will use the resting-state as an experimental paradigm for EEG. Recall that the resting-state is a constant condition without stimuli or behavioral events (Snyder
Our research design is a 2 (resting-state: eyes-open vs. eyes-closed) x 3 (time-samples of measurement: 10 sec., 10 sec., 10 sec.) fully repeated-measures design of the resting-state paradigm. Based on previous literature, we should find that the eyes-closed condition has greater neural activity than the eyes-open condition (Barry et al., 2007). If LMM is providing more precision on any measured effect (viz., the main effects or the interaction term), then we should expect to find smaller standard errors per term than in the RMANOVA analysis of these same data. Of note, because LMM uses a regression-based modeling while RMANOVA uses an ANOVA modeling of the data, we will report the RMANOVA results first and then convert the RMANOVA results into repeated-measures OLS multiple regression (cf. Pedhazur, 1982; cf. Tabachnick & Fidell, 2013, pp. 156-158) to compare the RMANOVA coefficients directly to the LMM coefficients.

Method

Participants

Seven students (3 Female, $M_{age} = 29.71$ years, $SD_{age} = 6.75$) from San Francisco State University participated in data collection. San Francisco State University’s Institutional Review Board approved the data for secondary analysis.

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1 From 1 condition (60 sec.) there are 3 samples: 1st 20 sec. a 10 sec. sample, 2nd 20 sec. a 10 sec. sample, and 3rd 20 sec. a 10 sec. sample.
**Procedures**

Each participant pre-scheduled an appointment to visit the lab. The participant sat in a chair, while the researcher placed EEG electrodes on the head of the participant. The researcher instructed the participant to close their eyes for 60 seconds and open their eyes for 60 seconds. The eyes-closed or eyes-open conditions were counterbalanced between participants. This procedure took approximately fifteen minutes. After the task, the electrodes were removed and the participant received gratitude from the researcher for engaging in the procedure.

**EEG Recording**

Continuous EEG was recorded from three disposable, vinyl electrodes (BIOPAC Systems, Inc., EL503) placed on the heads of the participant, according to the International 10–20 Electrode Placement System. Three electrodes were placed on the head: one over the midline site (Cz), one on the right mastoid bone as reference, and one on the forehead as ground. Before electrode application, the skin of the participant was cleaned with skin cleanser and alcohol wipe on the forehead and mastoid areas. Each of the electrodes had a water-soluble, conductive gel (BIOPAC Systems Inc., GEL1) applied to them to ensure proper connectivity. Tape was applied on top of electrodes to keep them in place. A Lycra cap, resembling a swimming cap, was placed on the head of the participant to secure all electrodes. EEG was amplified using the BIOPAC MP36 data acquisition system (BIOPAC Systems Inc., MP36). A lead set (BIOPAC Systems Inc., SS2L) with clips connected the electrodes to the data acquisition system.

EEG data were digitally recorded on a separate computer (Asus Sonicmaster laptop) using Acqknowledge 4.3 software. EEG data processing was performed using Acqknowledge and Microsoft Excel. Artifacts were inspected visually and rejected manually. Data were sampled at
1000 Hz and filtered online with a 0.01 to 30 Hz bandpass. All electrode impedances were kept below 20 kΩ. The waveform was bandpass filtered offline such that the alpha band was 8 Hz to 13 Hz, beta band was 13 Hz to 30 Hz, delta band was 0.5 Hz to 4 Hz, and theta band was 4 Hz to 8 Hz. For each 10 second sample from the EEG waveform, the standard deviation was the measurement of each amplitude.

Statistical Analysis

A repeated-measures design observed participants over time with measures taken for each participant under different conditions (Bagiella et al., 2000). This is a 2 x 3 research design per each EEG frequency band. The independent variable was the eyes-closed or eyes-open conditions that were counterbalanced between participants. The waveform in each condition is sampled for the first 10 seconds, second 10 seconds, and third 10 seconds. The dependent variable is the amplitude in microvolts (µV). Each amplitude is the standard deviation which is a measurement of the variability of the selected waveform. The same data set has two statistical analyses. First a repeated-measures ANOVA and second a linear mixed model to compare results. Four repeated-measures ANOVA models and four linear mixed models analyzed the data from each of the four EEG frequency bands. Given the small sample size, although we ran outlier screenings, we kept the outliers in the models (because univariate outlier detection is unreliable at n < 30).

Results

All analyses were conducted using the integrated developer environment R (R Core Team, 2022). We used the ez package (Lawrence, 2016) for the repeated-measures analyses of variance and the lmerTest package (Kuznetsova et al., 2017) for the linear mixed effects.
analyses. We ran parallel analyses between the repeated-measures ANOVAs and the LMMs to compare the comparable coefficients between the models. Because our sample size is quite small, we report $p$-values for convenience, but we understand that the sample is underpowered to detect statistical departures from presumed chance variation. Instead, we focus on the standard error coefficients and the beta-weights (as converted mean differences) as our main standards of comparison.

**Repeated-Measures Analysis of Variance**

We conducted four repeated-measures ANOVAs, one on each of the four EEG frequencies (alpha, beta, delta, and theta). For each ANOVA model, we used the default settings in the ez package to partition variance and estimate all effects.

**Alpha Frequency.** A repeated-measures ANOVA (including outliers) found a main effect for condition, with a significant difference between the eyes-closed condition ($M = 6.61, SE = 0.45$), and eyes-open condition ($M = 3.8, SE = 0.22$), for amplitudes in the alpha frequency, $F(1, 6) = 26.15, p = 0.002, \eta^2_G = 0.444$. There were no significant main effect for the time of the samples for amplitudes in the alpha frequency, $F(2, 12) = 0.05, p = 0.955, \eta^2_G = 0.001$, and no significant differences between the interaction of conditions and time of the samples for amplitudes in the alpha frequency, $F(2, 12) = 0.631, p = 0.549, \eta^2_G = 0.004$.

**Beta Frequency.** A repeated-measures ANOVA (including outliers) found a main effect for condition with a significant difference between the eyes-closed condition ($M = 5.87, SE = 0.35$), and eyes-open condition ($M = 4.4, SE = 0.28$), for amplitudes in the beta frequency, $F(1, 6) = 28.49, p = 0.002, \eta^2_G = 0.212$. There were no significant main effect for the time of the samples for amplitudes in the beta frequency, $F(2, 12) = 0.37, p = 0.696, \eta^2_G = 0.001$, and no significant
differences between the interaction of conditions and time of the samples for amplitudes in the beta frequency, $F(2, 12) = 1.06, p = 0.377, \eta^2_G = 0.004$.

**Delta Frequency.** A repeated-measures ANOVA including outliers found no main effect for condition with no significant differences between the eyes-closed condition ($M = 7.26, SE = 0.34$), and eyes-open condition ($M = 6.62, SE = 0.37$), for amplitudes in the delta frequency, $F(1, 6) = 2.25, p = 0.184, \eta^2_G = 0.042$. There were no significant main effect for the time of the samples for amplitudes in the delta frequency, $F(2, 12) = 2.46, p = 0.127, \eta^2_G = 0.051$, and no significant differences between the interaction of conditions and time of samples for amplitudes in the delta frequency, $F(2, 12) = 1.76, p = 0.213, \eta^2_G = 0.023$.

**Theta Frequency.** A repeated-measures ANOVA without outliers found a main effect for condition with a significant difference between eyes-closed ($M = 6.29, SE = 0.36$), and eyes-open condition ($M = 4.17, SE = 0.2$), for amplitudes in the theta frequency, $F(1, 6) = 21.56, p = 0.004, \eta^2_G = 0.401$. There were no significant main effect for the time of the samples for amplitudes in the theta frequency, $F(2, 12) = 0.15, p = 0.862, \eta^2_G = 0.002$, and no significant differences between the interaction of conditions and time of the samples for amplitudes in the theta frequency, $F(2, 12) = 0.361, p = 0.705, \eta^2_G = 0.003$.

*Linear Mixed Effects Regression*

We conducted four linear mixed-effects models (LMMs), one on each of the four EEG frequencies (alpha, beta, delta, and theta). For each LMM, we treated participants as random intercepts along the amplitude and used the Satterthwaite adjustment to the $p$-values per effect. For all effects with greater than 3 levels, the *lmerTest* package (Kuznetsova et al., 2017) in R created a dummy code that treated the first level of each variable as the comparison case. As
before with the RMANOVA results, we understand that the sample is underpowered to detect statistical departures from presumed chance variation. Instead, we focus on the standard error coefficients and the beta-weights (as converted mean differences) as our main standards of comparison. The random intercepts have been visualized in Figures 1-4 so that a reader can understand how much per-person variability there is on each frequency.

**Alpha Frequency.** The LMM results showed that the random intercept did not account for significant variability of the amplitudes in the alpha frequency ($b = 6.64$, $SE = 0.64$), $t(0.58) = 10.33$, $p = 0.161$, ICC$_{participants} = 0.645$. Nonetheless, the ICC was large at 64.5%. Thus, the small sample size may be creating a Type II error. Considering the fixed slope for the conditions variable, the eyes-closed condition did not significantly differ from the eyes-open condition for amplitudes in the alpha frequency ($b = -2.82$, $SE = 0.54$), $t(0.61) = -5.21$, $p = 0.227$. Considering the fixed slope for the time of sampling variable, the second sample did not significantly differ from the first for amplitudes in the alpha frequency ($b = 0.04$, $SE = 0.54$), $t(0.61) = 0.08$, $p = 0.955$. Considering the fixed slope for the time of sampling variable, the third sample did not significantly differ from the first for amplitudes in the alpha frequency ($b = -0.12$, $SE = 0.54$), $t(0.61) = -0.22$, $p = 0.878$. Considering the fixed slope for the interaction of the eyes-open condition compared to the eyes-closed condition, in the second sample versus the first sample, there is no significant difference for amplitudes in the alpha frequency ($b = -0.23$, $SE = 0.77$), $t(0.61) = -0.3$, $p = 0.834$. Considering the fixed slope for the interaction of the eyes-open condition compared to the eyes-closed condition, in the third sample versus the first sample, there is no significant difference for amplitudes in the alpha frequency ($b = 0.26$, $SE = 0.77$), $t(0.61) = 0.34$, $p = 0.814$. 
**Beta Frequency.** The LMM results showed that the random intercept did not account for significant variability of amplitudes in the beta frequency \((b = 5.9, SE = 0.58), t(0.22) = 10.24, p = 0.446, ICC_{participants} = 0.879\). Nonetheless, the ICC was large at 87.9%. Thus, the small sample size may be creating a Type II error. Considering the fixed slope for the conditions variable, the eyes-closed condition did not significantly differ from the eyes-open condition for amplitudes in the beta frequency \((b = -1.39, SE = 0.28), t(0.22) = -4.89, p = 0.516\). Considering the fixed slope for the time of sampling variable, the second sample did not significantly differ from the first sample for amplitudes in the beta frequency \((b = 0.05, SE = 0.28), t(0.22) = 0.17, p = 0.931\). Considering the fixed slope for the time of sampling variable, the third sample did not significantly differ from the first sample for amplitudes in the beta frequency \((b = -0.16, SE = 0.28), t(0.22) = -0.56, p = 0.811\). Considering the fixed slope for the interaction of the eyes-open condition compared to the eyes-closed condition, in the second sample versus the first sample, there was no significant difference for amplitudes in the beta frequency \((b = -0.32, SE = 0.4), t(0.22) = -0.79, p = 0.762\). Considering the fixed slope for the interaction of the eyes-open condition compared to the eyes-closed condition, in the third sample versus the first sample, there was no significant difference for amplitudes in the beta frequency \((b = 0.08, SE = 0.4), t(0.22) = 0.2, p = 0.921\).

**Delta Frequency.** The LMM results showed that the random intercept did not account for significant variability of amplitudes in the delta frequency \((b = 7.42, SE = 0.62), t(0.59) = 11.98, p = 0.142, ICC_{participants} = 0.604\). Nonetheless, the ICC was large at 60.4%. Thus, the small sample size may be creating a Type II error. Considering the fixed slope for the conditions variable, the eyes-closed condition did not significantly differ from the eyes-open condition for amplitudes in the delta frequency \((b = 0.03, SE = 0.55), t(0.63) = 0.05, p = 0.972\). Considering the fixed slope
for the time of sampling variable, the second sample did not significantly differ from the first sample for amplitudes in the delta frequency ($b = -0.31, SE = 0.55$), $t(0.63) = -0.57, p = 0.709$. Considering the fixed slope for the time of sampling variable, the third sample did not significantly differ from the first sample for amplitudes in the delta frequency ($b = -0.17, SE = 0.55$), $t(0.63) = -0.3, p = 0.832$. Considering the fixed slope for the interaction of the eyes-open condition compared to the eyes-closed condition, in the second sample versus the first sample, there was no significant difference for amplitudes in the delta frequency ($b = -1.0, SE = 0.78$), $t(0.63) = -1.28, p = 0.502$. Considering the fixed slope for the interaction of the eyes-open condition compared to the eyes-closed condition, in the third sample versus the first sample, there was no significant difference for amplitudes in the delta frequency ($b = -0.99, SE = 0.78$), $t(0.63) = -1.27, p = 0.505$.

**Theta Frequency.** The LMM results showed that the random intercept did not account for significant variability of amplitudes in the theta frequency ($b = 6.27, SE = 0.53$), $t(0.44) = 11.87, p = 0.219$, $IC_{participants} = 0.641$. Nonetheless, the ICC was large at 64.1%. Thus, the small sample size may be creating a Type II error. Considering the fixed slope for the conditions variable, the eyes-closed condition did not significantly differ from the eyes-open condition for amplitudes in the theta frequency ($b = -2.01, SE = 0.45$), $t(0.46) = -4.5, p = 0.325$. Considering the fixed slope for the time of sampling variable, the second sample did not significantly differ from the first sample for amplitudes in the theta frequency ($b = 0.04, SE = 0.45$), $t(0.46) = 0.1, p = 0.948$. Considering the fixed slope for the time of sampling variable, the third sample did not significantly differ from the first sample for amplitudes in the theta frequency ($b = 0.02, SE = 0.45$), $t(0.46) = 0.05, p = 0.973$. Considering the fixed slope for the interaction of the eyes-open condition compared to the eyes-closed condition, in the second sample versus the first sample, there was no
significant difference for amplitudes in the theta frequency ($b = -0.3$, $SE = 0.63$), $t(0.46) = -0.47$, $p = 0.776$. Considering the fixed slope for the interaction of the eyes-open condition compared to the eyes-closed condition, in the third sample versus the first sample, there was no significant difference for amplitudes in the theta frequency ($b = -0.02$, $SE = 0.63$), $t(0.46) = -0.03$, $p = 0.986$.

Comparison of RMANOVA and LMM Results

Because the RMANOVA and LMM results above are in different metrics (but ultimately comparable), we ran an (repeated measures) ordinary least-squares (RM-OLS) multiple regression on the RMANOVA ANOVA results to display them as slope coefficients with the same default dummy coding as the LMMs. Tables 1-4 display all coefficients between the OLS (aka RMANOVA) and LMM models by each EEG frequency: alpha (Table 1), beta (Table 2), delta (Table 3), and theta (Table 4).

**Table 1. LMM vs RM-OLS (Alpha Frequency)**

<table>
<thead>
<tr>
<th>Alpha Frequency</th>
<th>LMM $b$</th>
<th>LMM $SE$</th>
<th>RM-OLS $b$</th>
<th>RM-OLS $SE$</th>
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</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>$b = 6.64$</td>
<td>$SE = 0.64$</td>
<td>$b = 6.64$</td>
<td>$SE = 0.64$</td>
</tr>
<tr>
<td>Conditions</td>
<td>$b = -2.82$</td>
<td>$SE = 0.54$</td>
<td>$b = -2.82$</td>
<td>$SE = 0.91$</td>
</tr>
<tr>
<td>(Eyes-Closed/Eyes-Open)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time of Sampling</td>
<td>$b = 0.04$</td>
<td>$SE = 0.54$</td>
<td>$b = 0.04$</td>
<td>$SE = 0.91$</td>
</tr>
<tr>
<td>(2nd Time vs. 1st Time)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time of Sampling</td>
<td>$b = -0.12$</td>
<td>$SE = 0.54$</td>
<td>$b = -0.12$</td>
<td>$SE = 0.91$</td>
</tr>
<tr>
<td>(3rd Time vs. 1st Time)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 1. Beta coefficients and standard errors for Linear Mixed Models versus Repeated-Measures Ordinary Least Squares.

<table>
<thead>
<tr>
<th></th>
<th>LMM b</th>
<th>LMM SE</th>
<th>RM-OLS b</th>
<th>RM-OLS SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interaction of Conditions vs.</td>
<td>b = -0.23</td>
<td>SE = 0.77</td>
<td>b = -0.23</td>
<td>SE = 1.29</td>
</tr>
<tr>
<td>Time of Sampling (EC/EO in 2nd</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time vs. 1st Time)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interaction of Conditions vs.</td>
<td>b = 0.26</td>
<td>SE = 0.77</td>
<td>b = 0.26</td>
<td>SE = 1.29</td>
</tr>
<tr>
<td>Time of Sampling (EC/EO in 3rd</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Time vs. 1st Time)</td>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

For the alpha frequency, we compared the unstandardized beta coefficients ($b$) and standard errors ($SE$), between the LMM and OLS models. Results for the intercept (random for LMM and fixed for RM-OLS) along the amplitudes in the alpha frequency for LMM ($b = 6.64$, $SE = 0.64$) and OLS ($b = 6.64$, $SE = 0.64$) had the same beta coefficient and standard error. Considering the slope for the conditions variable, when comparing the eyes-closed condition and eyes-open condition, for amplitudes in the alpha frequency, LMM ($b = -2.82$, $SE = 0.54$) and OLS ($b = -2.82$, $SE = 0.91$) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the time of sampling variable, when comparing the second sample and the first sample, for amplitudes in the alpha frequency. LMM ($b = 0.04$, $SE = 0.54$) and OLS ($b = 0.04$, $SE = 0.91$) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the time of sampling variable, when comparing the third sample and the first sample, for amplitudes in the alpha frequency. LMM ($b = -0.12$, $SE = 0.54$) and OLS ($b = -0.12$, $SE = 0.91$) had the same beta coefficient and a smaller standard error for LMM than OLS.
Considering the slope for the interaction, when comparing the eyes-open condition and eyes-closed condition, in the second sample versus the first sample, for amplitudes in the alpha frequency. LMM \((b = -0.23, SE = 0.77)\) and OLS \((b = -0.23, SE = 1.29)\) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the interaction, when comparing the eyes-open condition and eyes-closed condition, in the third sample versus the first sample, for amplitudes in the alpha frequency. LMM \((b = 0.26, SE = 0.77)\) and OLS \((b = 0.26, SE = 1.29)\) had the same beta coefficient and a smaller standard error for LMM than OLS.

### Table 2. LMM vs RM-OLS (Beta Frequency)

<table>
<thead>
<tr>
<th>Beta Frequency</th>
<th>LMM (b)</th>
<th>LMM (SE)</th>
<th>RM-OLS (b)</th>
<th>RM-OLS (SE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>(b = 5.90)</td>
<td>(SE = 0.58)</td>
<td>(b = 5.90)</td>
<td>(SE = 0.58)</td>
</tr>
<tr>
<td>Conditions</td>
<td>(b = -1.39)</td>
<td>(SE = 0.28)</td>
<td>(b = -1.39)</td>
<td>(SE = 0.82)</td>
</tr>
<tr>
<td>(Eyes-Closed/Eyes-Open)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time of Sampling (2nd Time vs. 1st Time)</td>
<td>(b = 0.05)</td>
<td>(SE = 0.28)</td>
<td>(b = 0.05)</td>
<td>(SE = 0.82)</td>
</tr>
<tr>
<td>Time of Sampling (3rd Time vs. 1st Time)</td>
<td>(b = -0.16)</td>
<td>(SE = 0.28)</td>
<td>(b = -0.16)</td>
<td>(SE = 0.82)</td>
</tr>
<tr>
<td>Interaction of Conditions vs. Time of Sampling</td>
<td>(b = -0.32)</td>
<td>(SE = 0.40)</td>
<td>(b = -0.32)</td>
<td>(SE = 1.15)</td>
</tr>
</tbody>
</table>
For the beta frequency, we compared the unstandardized beta coefficients ($b$) and standard errors ($SE$), between the LMM and OLS models. Results for the intercept (random for LMM and fixed for RM-OLS) along the amplitudes in the beta frequency for LMM ($b = 5.9$, $SE = 0.58$) and OLS ($b = 5.9$, $SE = 0.58$) had the same beta coefficient and standard error. Considering the slope for the conditions variable, when comparing the eyes-closed condition and eyes-open condition, for amplitudes in the beta frequency. LMM ($b = -1.39$, $SE = 0.28$) and OLS ($b = -1.39$, $SE = 0.82$) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the time of sampling variable, when comparing the second sample and the first sample, for amplitudes in the beta frequency. LMM ($b = 0.05$, $SE = 0.28$) and OLS ($b = 0.05$, $SE = 0.82$) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the time of sampling variable, when comparing the third sample and the first sample, for amplitudes in the beta frequency. LMM ($b = -0.16$, $SE = 0.28$) and OLS ($b = -0.16$, $SE = 0.82$) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the interaction, when comparing the eyes-open condition and eyes-closed condition, in the second sample versus the first sample, for amplitudes in the beta frequency. LMM ($b = -0.32$, $SE = 0.4$) and OLS ($b = -0.32$, $SE = 1.15$) had the same beta coefficient and a smaller standard error.
for LMM than OLS. Considering the slope for the interaction, when comparing the eyes-open condition and eyes-closed condition, in the third sample versus the first sample, for amplitudes in the beta frequency. LMM ($b = 0.08, SE = 0.4$) and OLS ($b = 0.08, SE = 1.15$) had the same beta coefficient and a smaller standard error for LMM than OLS.

**Table 3. LMM vs RM-OLS (Delta Frequency)**

<table>
<thead>
<tr>
<th>Delta Frequency</th>
<th>LMM $b$</th>
<th>LMM $SE$</th>
<th>RM-OLS $b$</th>
<th>RM-OLS $SE$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>$b = 7.42$</td>
<td>$SE = 0.62$</td>
<td>$b = 7.42$</td>
<td>$SE = 0.62$</td>
</tr>
<tr>
<td>Conditions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Eyes-Closed/Eyes-Open)</td>
<td>$b = 0.03$</td>
<td>$SE = 0.55$</td>
<td>$b = 0.03$</td>
<td>$SE = 0.88$</td>
</tr>
<tr>
<td>Time of Sampling</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2nd Time vs. 1st Time)</td>
<td>$b = -0.31$</td>
<td>$SE = 0.55$</td>
<td>$b = -0.31$</td>
<td>$SE = 0.88$</td>
</tr>
<tr>
<td>Time of Sampling</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3rd Time vs. 1st Time)</td>
<td>$b = -0.17$</td>
<td>$SE = 0.55$</td>
<td>$b = -0.17$</td>
<td>$SE = 0.88$</td>
</tr>
<tr>
<td>Interaction of Conditions vs. Time of Sampling</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(EC/EO in 2nd Time vs. 1st Time)</td>
<td>$b = -1.00$</td>
<td>$SE = 0.78$</td>
<td>$b = -1.00$</td>
<td>$SE = 1.24$</td>
</tr>
<tr>
<td>Interaction of Conditions vs. Time of Sampling</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(EC/EO in 3rd Time vs. 1st Time)</td>
<td>$b = -0.99$</td>
<td>$SE = 0.78$</td>
<td>$b = -0.99$</td>
<td>$SE = 1.24$</td>
</tr>
</tbody>
</table>

*Table 3. Beta coefficients and standard errors for Linear Mixed Models versus Repeated-Measures Ordinary Least Squares.*
For the delta frequency, we compared the unstandardized beta coefficients \((b)\) and standard errors \((SE)\), between the LMM and OLS models. Results for the intercept (random for LMM and fixed for RM-OLS) along the amplitudes in the delta frequency for LMM \((b = 7.42, SE = 0.62)\) and OLS \((b = 7.42, SE = 0.62)\) had the same beta coefficient and standard error. Considering the slope for the conditions variable, when comparing the eyes-closed-condition and eyes-open condition, for the amplitudes in the delta frequency. LMM \((b = 0.03, SE = 0.55)\) and OLS \((b = 0.03, SE = 0.88)\) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the time of sampling variable, when comparing the second sample and the first sample, for the amplitudes in the delta frequency. LMM \((b = -0.31, SE = 0.55)\) and OLS \((b = -0.31, SE = 0.88)\) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the time of sampling variable, when comparing the third sample and the first sample, for the amplitudes in the delta frequency. LMM \((b = -0.17, SE = 0.55)\) and OLS \((b = -0.17, SE = 0.88)\) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the interaction, when comparing the eyes-open condition and eyes-closed condition, in the second sample versus the first sample, for the amplitudes in the delta frequency. LMM \((b = -1.00, SE = 0.78)\) and OLS \((b = -1.00, SE = 1.24)\) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the interaction, when comparing the eyes-open condition and eyes-closed condition, in the third sample versus the first sample, for the amplitudes in the delta frequency. LMM \((b = -0.99, SE = 0.78)\) and OLS \((b = -0.99, SE = 1.24)\) had the same beta coefficient and a smaller standard error for LMM than OLS.
Table 4. LMM vs RM-OLS (Theta Frequency)

<table>
<thead>
<tr>
<th>Theta Frequency</th>
<th>LMM $b$</th>
<th>LMM $SE$</th>
<th>RM-OLS $b$</th>
<th>RM-OLS $SE$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>$b = 6.27$</td>
<td>$SE = 0.53$</td>
<td>$b = 6.27$</td>
<td>$SE = 0.53$</td>
</tr>
<tr>
<td>Conditions (Eyes-Closed/Eyes-Open)</td>
<td>$b = -2.01$</td>
<td>$SE = 0.45$</td>
<td>$b = -2.01$</td>
<td>$SE = 0.75$</td>
</tr>
<tr>
<td>Time of Sampling (2nd Time vs. 1st Time)</td>
<td>$b = 0.04$</td>
<td>$SE = 0.45$</td>
<td>$b = 0.04$</td>
<td>$SE = 0.75$</td>
</tr>
<tr>
<td>Time of Sampling (3rd Time vs. 1st Time)</td>
<td>$b = 0.02$</td>
<td>$SE = 0.45$</td>
<td>$b = 0.02$</td>
<td>$SE = 0.75$</td>
</tr>
<tr>
<td>Interaction of Conditions vs. Time of Sampling (EC/EO in 2nd Time vs. 1st Time)</td>
<td>$b = -0.30$</td>
<td>$SE = 0.63$</td>
<td>$b = -0.30$</td>
<td>$SE = 1.06$</td>
</tr>
<tr>
<td>Interaction of Conditions vs. Time of Sampling (EC/EO in 3rd Time vs. 1st Time)</td>
<td>$b = -0.02$</td>
<td>$SE = 0.63$</td>
<td>$b = -0.02$</td>
<td>$SE = 1.06$</td>
</tr>
</tbody>
</table>

Table 4. Beta coefficients and standard errors for Linear Mixed Models versus Repeated-Measures Ordinary Least Squares.

For the theta frequency, we compared the unstandardized beta coefficients ($b$) and standard errors ($SE$), between the LMM and OLS models. Results for the intercept (random for LMM and fixed for RM-OLS) along the amplitudes in the theta frequency for LMM ($b = 6.27$, $SE = 0.53$) and OLS ($b = 6.27$, $SE = 0.53$) had the same beta coefficient and standard error. Considering the slope for the conditions variable, when comparing the eyes-closed condition and eyes-open
condition, for amplitudes in the theta frequency. LMM \((b = -2.01, SE = 0.45)\) and OLS \((b = -2.01, SE = 0.75)\) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the time of sampling variable, when comparing the second sample and the first sample, for amplitudes in the theta frequency. LMM \((b = 0.04, SE = 0.45)\) and OLS \((b = 0.04, SE = 0.75)\) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the time of sampling variable, when comparing the third sample and the first sample, for amplitudes in the theta frequency. LMM \((b = 0.02, SE = 0.45)\) and OLS \((b = 0.02, SE = 0.75)\) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the interaction, when comparing the eyes-open condition and eyes-closed condition, in the second sample versus the first sample, for amplitudes in the theta frequency. LMM \((b = -0.30, SE = 0.63)\) and OLS \((b = -0.30, SE = 1.06)\) had the same beta coefficient and a smaller standard error for LMM than OLS. Considering the slope for the interaction, when comparing the eyes-open condition and eyes-closed condition, in the third sample versus the first sample, for amplitudes in the theta frequency. LMM \((b = -0.02, SE = 0.63)\) and OLS \((b = -0.02, SE = 1.06)\) have the same beta coefficient and a smaller standard error for LMM than OLS.

**Discussion**

We were interested in comparing the results from repeated-measures ANOVA (RMANOVA) and Linear Mixed Modeling (LMM) as statistical techniques for the analysis of pre-processed EEG data. To accomplish the goals of the paper, we ran an ordinary least-squares (OLS) multiple regression on the RMANOVA results (to put these results in the larger regression framework, resulting in a [repeated-measures] RM-OLS) and compared the results as slope
coefficients with the same default dummy coding as the LMM models by each EEG frequency. (Importantly, we omitted the report of the degrees of freedom for the RM-OLS, which, for completeness, are adjusted for included terms, which is parallel to, but not the same as, the Greenhouse-Geisser adjustment in RMANOVA.)

Overall, the alpha frequency amplitudes had 5 standard errors that were different between the LMM and RM-OLS models, out of 6 total standard errors, but all the unstandardized slope coefficients were the same. The 5 standard errors were smaller for LMM than RM-OLS models. Overall, the beta frequency amplitudes had 5 standard errors that were different between the LMM and RM-OLS models, out of 6 total standard errors, but all the unstandardized slope coefficients were the same. The 5 standard errors were smaller for LMM than RM-OLS models. Overall, the delta frequency amplitudes had 5 standard errors that were different between the LMM and RM-OLS models, out of 6 total standard errors, but all the unstandardized slope coefficients were the same. The 5 standard errors were smaller for LMM than RM-OLS models. Overall, the theta frequency amplitudes had 5 standard errors that were different between the LMM and RM-OLS models, out of 6 total standard errors, but all the unstandardized slope coefficients were the same. The 5 standard errors were smaller for LMM than RM-OLS models. The RM-OLS multiple regression interpreted the RMANOVA results to demonstrate that in regression the error variance was distributed between the unweighted beta coefficient and standard error.

As noted in the Introduction, because of this more specific organization of the data and further partitioning of variance in LMM, in the majority of cases, the standard error should be smaller in LMM compared to RM-OLS models. We confirmed this expected result for all EEG
frequencies and in almost all cases. More specifically, because the sample size stayed the same across the comparisons, the error variance has changed between the LMM and RM-OLS models (of the same data), which shows that the error variance itself is further partitioned in a manner that it is not in RM-OLS (and OLS more broadly).

Implications

The sample size of our data is too small to establish any statistically significant effects using null hypothesis statistical tests. However, the evidence of precision from the lower standard errors in LMM as compared to RM-OLS models revealed itself even with the small sample size. Consequently, the reduction in error variance is clear, even without relying on \( p \)-values. Psychophysiology, cognitive, and neurology research should still be encouraged to use LMM to attribute person variability because such data analyses will reduce error variance by appropriately assigning it to individual response profiles on the electrical activity and tasks under investigation. Researchers can find the differences in the amplitudes of neural oscillations during resting-state conditions. By deploying LMM, they can immediately organize (pre-processed) data at a level by the variability that people have on the task. By characterizing the overall pattern, researchers can track better where human participants are going to go knowing the random intercept. LMM makes the data analysis less statistically noisy, hence there is more precision. Regarding statistical power for LMM, the levels of the grouping variable in any random effect has a relationship to statistical power (Westfall et al., 2015) with authors suggestion minimum of 5 levels to the grouping variable (7 participants) (cf. Bolker, 2015) up to 20 levels (Singmann & Kellen, 2019). When used with larger samples sizes and at least the minimum groupings levels, LMM has the potential to detect to uncover more potential
statistically significant effects (while controlling for Type I error inflation, via the Satterthwaite or Kenward-Roger corrections) and simultaneously avoid the Type II errors that would otherwise happen using RMANOVA (or RM-OLS) for psychophysiology, cognitive, and neurology research.
References


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Appendix A: Random Effect Line Graphs

Figure 1. Random Effect of Participants (Alpha Frequency)

Figure 1. Time Samples by Amplitudes for each condition (Top: Eyes-Open Bottom: Eyes-Closed) and each participant.
Figure 2. Random Effect of Participants (Beta Frequency)

Figure 2. Time Samples by Amplitudes for each condition (Top: Eyes-Open, Bottom: Eyes-Closed) and each participant.
Figure 3. Random Effect of Participants (Delta Frequency)

Figure 3. Time Samples by Amplitudes for each condition (Top: Eyes-Open, Bottom: Eyes-Closed) and each participant.
Figure 4. Random Effect of Participants (Theta Frequency)

Figure 4. Time Samples by Amplitudes for each condition (Top: Eyes-Open, Bottom: Eyes-Closed) and each participant.