THE EFFECT OF QUANTITATIVE EASING TO THE U.S. ECONOMY:
UNDER TVP-VAR APPROACH

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ABSTRACT

This paper analyses the dynamic macroeconomic effect of the 4 round Quantitative Easing (QE) adopted by the Federal Reserve (Fed) that started in November 2008. The paper uses Time Varying Parameter Vector AutoRegression (TVP-VAR) method to study the relationship among the monetary base, industrial production and inflation rate. The results show that monetary base has distinct pulling effect on industrial production, but not on inflation rate; yet, industrial production shock has no obvious effect on monetary base increasing, but effect on inflation rate is decreasing in one year lag; inflation shock has negative effect on monetary base and positive effect on industrial production, both these two effect are stable over the sample period. Accordingly, my results imply that the U.S. QE experiment was successful in stimulating economic activity, it does not increase the inflation rate in short run, but the long run effect on inflation is not determined. By giving interpretation of these results, this paper proposes that while the QE has helped support the economy, it still carries some risks.

Keywords: quantitative easing, macroeconomics, TVP-VAR method
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Introduction

On November 25, 2008, the Federal Reserve (Fed) launched the first round of quantitative easing buying $500 billion in Mortgage-Backed Securities (MBS) and $100 billion debt of Fannie Mae, Freddie Mac and the Federal Home Loan Banks (Mullins, 2008, para. 2). From that time, the Federal Open Market Committee (FOMC) introduced the unconventional policy to booster the weak financial market, instead of using the federal funds rate target which was the main market operating target in recent years. This change of monetary policy was due to short-term nominal interest rates have been approached zero since November 2008 and the condition got worse, the Fed needed to ease monetary policy to stimulate the fragile economy and to fight deflationary pressure.

Through above policy implementation, we can see Quantitative Easing (QE) is a non traditional monetary policy used by the Fed to booster economy by significantly increasing money supply (Kapetanios, Mumtaz, Stevens, & Theodoridis, 2012). In details, the policy of QE involved in buying large quantities of long-term government securities and part of private assets using Fed’s money, in order to increasing capital liquidity in the market, creating more economic activity, achieving the inflation target and boosting spending (Kapetanios et al., 2012). Through QE program, the Fed dumped total $2.284 trillion to increase the monetary base from August 1, 2008 to June 14, 2013. The injection of large amount of money into the financial market would increase the market confidence with ample liquidity. The growth in reserves would cause financial assets more attractive for investors to buy, which resulting in an increase in stock prices, depreciation of the dollar, and increasing bank lending (Kimura, Kobayashi, Muranaga, & Ugai, 2003). Moreover, the Fed commitment of raising the reserves might have
positive effects on business and consumers” expectations on the future economy and prices (Kimura et al., 2003). At last, the QE program is cumulative which means a new program is adding on the previous program, by this means the Fed can gradually conduct the QE policy in a moderate pace.

However, some critics have questioned whether Fed’s actions have truly helped support the economy over the past five years, given economic growth remains sluggish. Money supply changes by Fed can have a large effect on economic trend shifts, positively or negatively. Except booms, many economists argue that busts and bubble are also caused by Fed money supply manipulation. An intense debt about QE program whether it will trigger inflation later and whether it should be extended, along with its effectiveness come up with national concern. The paper is motivated by several empirical evidences indicated QE purchasing has impact on economic activities, details are in the literature review section.

So far, the empirical evidence associated with the effects of quantitative easing shocks in “liquid trap” is rather scarce, and researchers hold different opinions on the effectiveness of this policy to the real economy. An apparent reason might be that the U.S. economy has been in such a condition shortly so the time period is short, the sample data used in research is limited sample. Furthermore, most of the studies focus on various interest rates to estimate the effect. They seldom study the effects on other macro variables such as industrial production and inflation. Five years have been passed since the Fed first launched QE program, I attempt to quantify of quantitative monetary easing measures by focusing on the money supply shock impact on the economy.

The paper is arranged as follows:
We began in section 1 with an overview of the feature of QE program adopted by Fed since financial crisis. Section 2 describes the transmission mechanism of monetary policy under liquidity trap conditions and the effects of monetary policy shocks. Section 3 provides literature review on empirical analysis. In Section 4, I first do the unit root test before making them stationary, and do the Johanson integration test to see if it has long run equilibrium, then estimate impulse response among variables through basic VAR model. Section 5 gives a hypothesis development, the theory method behind a proposed relationship between variables through theoretical viewpoint, including vector auto-regressive model (VAR), Markov chain Monte Carlo (MCMC) method and time-varying parameter vector auto-regressive (TVP-VAR) model. Section 6 does empirical analysis through TVP-VAR to evaluate the effects of the monetary base shock during financial crisis, and describe the estimation procedure. The results part in section 7 discusses the impulse response function to analyze the how the quantitative easing impact positively on economic activity but less on inflation, and I make a comparison of basic VAR and TVP-VAR model. Section 8 presents conclusion and related concern.
Recent Developments of Monetary Policy in United States

Recent Monetary Policy

The collapse of housing bubble in 2007 was the beginning of a long economic recession in the U.S.. The Fed reacted by lowering policy rate target and discount rate. At end of 2008, federal funds rate has approached 0.25% from 5% on September 2007, with little room for further reducing (Labonte, 2014). In December 2008, the assistance from Fed has reached $1.6 trillion and a near $108 billion given in direct support (Labonte, 2014). But liquidity problems persisted. Liquidity provided to the banking system has not taken an effect on rest of the financial system. This circumstance prevented traditional monetary transmission mechanism to activate the economy working correctly, so at the same time Fed decided to provide additional monetary policy stimulus by adopting first quantitative monetary easing in November 2008 (Labonte, 2014).

The Fed has launched four rounds of QE since the financial crisis 2008. QE1 was formally launched in November 2008. After QE1, Fed enacted QE2 from November 2010 and end in June 2011, with the total purchase of 600 billion dollars of treasuries that between 2.5 and 10 years maturities in length (Labonte, 2014). QE3 is from September 2012 till now, planning to purchase 400 billion MBS every month, keeping federal funds rate at 0%-0.25%, and this period lasts at least to 2015. QE4 was announced in January 2013 that purchases 450 billion treasuries every month. Along with QE3 400 billion MBS purchase per month, Fed’s purchasing can add up to 850 billion every month (Labonte, 2014). Fed also announced that long run objective of inflation rate remain below 2.5% within one or two years, and unemployment rate will keep at 6.5% (Monetary Policy Report, 2014).
Transmission Mechanism of Unconventional Monetary Policy

There are mainly three instruments for Fed to conduct monetary policy: the first is open market operations, the second is discount rate and the least is reserve requirement. All of them can expand or contract money and credit (Labonte, 2014). Open market operations is the primary instrument that Fed using to buy or sell government securities to affect the short-term nominal interest rate. This action will affect the bank reserve base and influence the credit expansion of depository institutions (Labonte, 2014). The traditional monetary policy can potentially affect economy through two types of channels: interest rate channel and credit channel. Through open market operations, Fed can affect variety of asset price such as stock price and exchange rate (Fawley & Neely, 2013). Lower interest rate can directly encourage household and business borrowing, meanwhile higher stock price means the higher consumer wealth and stock issuing are more attractive, these would increase the consumption and investment. The depreciation of exchange rate will make domestic goods more competitive to foreign goods. Monetary policy can reduce financial frictions that limit the borrowing activities, e. g. adverse selection and moral hazard through credit channel (Fawley & Neely, 2013). The lower interest rate and high stock price prevent fraudulent activities, which can lower the borrowing cost and would ultimately encourage economic activities.

However, if these two channels were the only monetary transmission that works effect, purchasing short-term securities cannot further stimulate economy when the short-term nominal interest rates approach zero (Kimura et al., 2013). The 2008 financial crisis revealed that open market operations alone is not sufficient. The Fed has to turn to unconventional monetary policies which are intend to direct lend to short-term money
market and purchase long-term asset to lower the long-term interest rate. During 2007 to 2009, direct lending through the discount window to increase the banking system liquidity along with the decline in policy rate dominated the monetary policy. But the unresponsive of aggregate demand to these massive money supply indicates that the economy has reached a “liquidity trap” (Labonte, 2014). After December 2008, Fed heavily involved in purchasing long-term securities to decrease real long-term interest rates.

There are three options mainly used by Fed to stimulate the economy when interest rate is near zero. First is maintaining short-term interest rate low, slightly above zero; the second is increasing reserve to expand bank credit creation; the third is the Fed directly lowering long term rates (Sellon, 2003).

Unconventional monetary policy transmission mechanism has three channels to affect the economy. The first is through expanding the quantity of reserve. Bank can make additional loan by using excess reserve, this will in turn push down mid-term and long-term interest rate in order to recover market confidence (Sellon, 2003). By providing huge amount of money into the market the risk premium would be lowered which may further stimulate economic activities. But the drawback is even with huge excess reserve, banks may still reluctant to lend it out.

The second is to lower long-term real interest rate, which can be declined in three ways. The first is the falling in expected policy rate path. While the interest rate is near zero, it would still have a stimulate effect on the economy. The FOMC committee uses forward commitment to maintain its policy target rate lower between 0% and 0.25% until at least middle 2015. The object is to induce maximum effects of zero short-term interest
rate which will in turn reflect on other interest rate to stimulate business and household borrowing. Because long-term interest rates is the average expected short-term rates plus its risk premium, the commitment to maintain the low policy rate longer to the future, properly to mid-2015, would lower expected short-term interest rates in order to lower long-term interest rate (Kimura et al., 2003). Fed believes businesses would be more likely to make long-term investment if they are confident that interest rates will remain low over the loan period (Labonte, 2014).

The second is increasing the inflation expectation. Fed manages financial marketing expectations on future Fed’s policy to strengthen the economy reviving (Labonte, 2014). Some worried that large volume of reserve adding to the banking system will trigger higher inflation as too much money chasing limited quantity of good. However, when federal funds rate keep stable, the enough of inflation would cause real interest rate go down which decrease borrowing cost and increase investment and consumption.

The third way to lower the long-term interest rate is the Fed targeting directly to reduce term premium. The Fed can also lower the long-term premium through asset purchases. Fed purchase large amount of risk securities will lower the risk premium on the risk securities, so long-term rate will decrease (Fawley & Neely, 2013). Thus through purchasing risk financial assets it will lower long-term interest rate and stimulate economy.

The third channel is through economy-wide credit allocation. Quantitative easing is often used to help an insolvent markets with deficient liquidity, such as the MBS market. Bad real estate performance caused huge losses for lenders and also losses for
investors on MBS and mortgage-related securities. By purchasing MBS, the Fed could allocate much money into the housing finance sector such as home sale and investment, which is the most in need of correction and stabilization in economy. Therefore, focusing large part of QE on housing-related finance would increase consumer wealth, business investment and other fields in economy which results in pulling up aggregate demand.

Because interest sensitive economic variables respond most to longer term interest rates like business and household, therefore purchasing large scale of long-term securities will increase the price, so rate is declined, establishing a ceiling on the yield. As mentioned above, we know the decline of real long-term interest rates means decreasing in borrowing cost. Business investment and consumer borrowing will respond actively if cost down and reward high. Thus, through these Large-Scale Asset Purchases (LSAPs) program, the Fed can increase the aggregate demand and influence the growth of GDP and employment rate.
Literature Review

Nelson (2002) discovered that money expansion positively affect the total output of the United States and United Kingdom when controlling the short-term real interest rate. Some studies have concluded that quantitative monetary easing appears to work effectively by reducing the risk long-term assets and increasing supply of safe liquid assets such as bank reserve. Gagnon, Raskin, Remache, and Sack (2010) estimate the impact of the LSAPs on the 10-year government bond yield spread. They found the overall size of the 10-year term premium has been reduced by these purchases, especially the yields on riskier Government-Sponsored Enterprise (GSE) and MBS securities. Chung, Laforte, Reifschneider, and Williams (2011) found that asset purchases policy by the Fed has alleviated the macroeconomic costs of the liquidity trap in the U.S. by conducting several structural and time series models. They also found the huge expansion of the Fed’s balance sheet since late 2008 lower the unemployment rate comparing to not adopt the LSAPs and QE may probably avoid the economy falling into deflation.

Some papers treat the untraditional monetary policy effect more explicitly in running the VAR. For example, Liu and Mumtaz (2012) run a change-point VAR model by using the Federal Funds Rate, Money Supply M2 and other variables to examine the influence of macroeconomic shocks during the financial crisis in the U.S.. They found the Fed's LSAPs decreased the 10-year interest rate spread by 90 basis points from 2008. Regard to macroeconomic aspect, LSAP program contributed to lowering the unemployment rate by about 0.7% while boosting inflation by about 1%. They also extended VAR model by including various variables, e.g. stock prices of S&P 500.
Because of the insufficient data and information to build a sustained monetary policy during the financial crisis period, the relevant studies for the effect of quantitative easing to the U.S. economy is relatively scarce. There are much more research done on the Japanese case as Bank of Japan has introduced unconventional monetary policy since 2001 with a near zero short-term interest rates and monetary base huge expansion year after year. Baig (2002) focus on Japan’s case to illustrate the massive money injection to the economy has contributed to increasing in Japan’s price level and output with the interest rate as a separate variable in the model. By running a Bayesian VAR, Kimura et al. (2003) concluded that while the quantitative monetary policy had taken effect on inflation before, but it would not lead to rise in inflation at zero interest rates condition. This study found some evidence that monetary base growth does not cause higher inflation. This observation was consistent with Japan’s recent low inflation situation. They also found that there still some room remaining for further providing funding to the economy in order to recover from the recession. Thus, they advocate a larger quantitative expansion stimulations at zero interest rate.

Finally, Berkmen (2012) gave a broader view by using several aspect macroeconomic variables such as real GDP growth rate, the inflation rate, term spread, nominal exchange rate to introduce monetary easing measures in the regressions without imposing transmission channel to trace their impact on economic activity. The VAR regression indicated that an unexpected increase in the Current Account Balances at the Bank of Japan has helped support economic activity. But this quantitative impact on inflation is weaker which reflects Japan’s stable inflation expectations and relatively flat Phillips curve. The variance decompositions showed that monetary policy effect on
economy revives relative to other variables included in the regressions to the variability of output and inflation is small, usually less than 10%.

The above quantitative monetary policy studies on macroeconomic variables using different types of VAR give me some idea about the relationship between monetary base growth and other macroeconomic variables such as the industrial production and inflation. But even the Kimura et al. (2003) using time changing coefficient in VAR model to estimate the changing of monetary base to the economy, the variance of structure shock is assumed to be constant which may lead to biases in estimation. According to the research on these literature, this paper use a TVP-VAR model to do the estimation, therefore the parameters and the disturbance variances are all changing over time.
Basic VAR Model

From Sims (1980) introduced Vector Autoregression (VAR) model, a new macroeconometric framework, it has been broadly used not only in time series analysis but many other fields as a basic econometric tool. A VAR is a sequence of linear equations in a matrix form with each vector variable can be explained in one structure equation; each vector variable is expanded based on its own lag values and current and lag values of other vector variables. The response of macroeconomic variables to the monetary policy innovation can be easily identified and analyzed by simple VAR (Bernanke, Boivin, & Eliasz, 2005). But this approach does not lack of criticism. Although it has strong capability in system analysis, the fixed constant parameter assumption impose a restriction on the wide of model utilizing. VAR model only can include a small number of variables crucial for analyzing the monetary policy, which so it only gives a simple interpretation to the model (Kimura et al., 2003). As come to 21 century, VAR model gradually evolutes to the direction of structure type, nonlinear, spatial econometrics and Bayesian statistical technology.

In order to introduce the TVP-VAR, I do the basic VAR analysis first to give a general idea about the monetary base shock to the industrial production index and inflation rate which the same variables I used in the TVP-VAR in section 6.

At the beginning, it is important to know the time series data are stationary or not, that means whether their mean, variance and covariance would not be changed over time. Thus unit root test is always conducted first to check the data’s stationary. If it is nonstationary then next step is to determine whether it follow a deterministic or stochastic trend. If deterministic, we can estimate the VAR with no problem. If
stochastic trend, then I will to apply Johanson cointegration test to see whether there is a long equilibrium relationship among nonstationary variables. If it is cointegrated I need to estimate a VAR-VEC model. For my sample data, series are not cointegrated, then I choose estimate the VAR by using stationary data after first difference.

Based on Schwarz criteria, I include three lags when doing VAR which is defined by Cholesky decomposition. The sequence is inflation, industrial production and monetary base. Based on Baig (2002), inflation assume to respond last to the impact of monetary policy, and the monetary base react strongly to the monetary shock.

Figure 1. Impulse response functions of simple VAR.

The sample period January, 2004 to April, 2014. DY is industrial production after first difference; DI is Inflation rate after first difference; DMB is monetary base after first difference. The solid line shows a 12 month response to a standard deviation shock, the dashed lines show ±2 standard deviation bands.
The above figure shows a impulse response functions of simple VAR. The graphs trace the effect of a one-time variable shock to other variables in the period January 2004 to April 2014, the same sample data used in TVP-VAR (see section 6). First, the inflation rate first responds negatively and then the response keeps close to zero to the monetary base innovations, and with the periods last to 12, the range of the response volatility shrink. The reaction of industrial production to the monetary base shock starts positive and then negative within first 4 period, and last relatively flat comparing to the inflation response.
Time-Varying Parameter VAR (TVP-VAR) with Stochastic Volatility

Literature Review of TVP-VAR

In recent 10 years, there has been an increasing interest in time-varying parameter vector autoregressive (TVP-VAR) modeling, especially in analyzing the effect of monetary policy and predicting macroeconomic development since Primiceri (2005) using the US economy data in TVP-VAR model that allows intercept, VAR parameter, variance and structure shock are all changing with the time. This specific theory was originally derived from Kim, Shephard, and Chib (1998) in a model of the stochastic volatility with comparison of ARCH model. Benati (2007) applied a TVP-VAR model to the UK economy and make a prediction of the inflation. Nakajima (2011) adopted a TVP-VAR model with stochastic volatility to test the effect of quantitative monetary policy specifically at the zero lower bound. He got a conclusion that the effect of quantitative easing to the other parts of the economy is not obvious. The primary reason he choose TVP-VAR instead of constant parameter VAR was Bank of Japan clearly switched its monetary policy due to financial market changing over time and economy is not stable within sample period such as from low interest rate period to QE program.

Based on the following facts that I select TVP-VAR instead of the basic VAR to do the empirical analysis: the change in monetary policy in different time period needs time changing parameters; we need specific response to each shock in the zero interest rates period rather than the average response of the shock to the economy (Kimura et al., 2011). It also has less theoretical data constraints and allows for a wider set of uncertainty which can show the time-varying nature of both financial market and economy. The monetary policy change from short-term policy rate targeting to QE
would lead to the shifts of transmission mechanism from the effect of interest rate on macroeconomic variables to an effect of money expansion. TVP-VAR can extract all the gradual or sudden changes due to types of shocks and show the interaction among variables over time. Also the TVP-VAR with stochastic volatility can best capture the underlying flexible and random nature of economy.

Compared to the unproblematic and relative smooth of the traditional monetary policy, unconventional policy is full of shock because of billions of dollars spending and purchasing. In this paper I am pragmatic about this problem and keep the focus on the aggregate economy and, by implication, on monetary base growth shocks is in line with the method that Nakajima (2011) used for the Japanese economy and my model is implemented by the TVP-VAR package from Nakajima (2011) with a slight modification.

Before defining the TVP-VAR, I start with a simple VAR. The algorithm list following is taken from Nakajima (2011).

\[ Ay_t = F_1 y_{t-1} + \ldots + F_s y_{t-s} + u_t \]

\( y_t \) is observed sample listed in an \( k \times 1 \) vector which represent a macroeconomic variable that capture both structure shock and systematic component, “\( A, F_1, \ldots, F_s \) are matrices coefficients, and \( u_t \) is a \( k \times 1 \) vector of innovations” (Nakajima, 2011, pp.123), with zero mean and variance matrix \( E(u_t u_t^\prime) = \Sigma u_t \).

Assuming \( A \) is lower-triangular, which is taken from the Nakajima (2011):

\[
A = \begin{pmatrix}
1 & 0 & \ldots & 0 \\
a_{21} & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
a_{k1} & \ldots & a_{k,k-1} & 1
\end{pmatrix}
\]
So rewrite to “\( y_t = B_1y_{t-1} + \cdots + B_s y_{t-s} + A^{-1}\Sigma \varepsilon_t \), where \( \varepsilon_t \sim N(0, I_k) \), \( B_i = A^{-1}F_i \)” (Nakajima, 2011, pp.123). The model can be rewritten as:

\[
\text{“ } y_t = x_t \beta + A^{-1}\Sigma \varepsilon_t \text{”}
\]

After calculating \( B_i \)'s, we get a \( k^2 \times 1 \) vector form \( \beta \), and \( x_t \) represents \( I_k \otimes (y_{t-1}, ..., y_{t-s}) \), where \( \otimes \) is kroneck symbol. All parameters in above equation are still constant, not time changing (Nakajima, 2011). But by allowing these parameters to change with time, this model can be extended to TVP-VAR.

Nakajima (2011) rewrites to TVP-VAR model as:

\[
\text{“ } y_t = x_t \beta_t + A_t^{-1}\Sigma_t \varepsilon_t \text{”}
\]

Assuming \( \beta_t, A_t \) and \( \Sigma_t \) are all changing with time, \( h_{jt} = \log \sigma^2_{jt} \). Nakajima (2011) assume that they are all random walk, he sets as follows: “\( \beta_{t+1} = \beta_t + u\beta_t, a_{t+1} = a_t + u_a, h_{t+1} = h_t + u_h \)” (pp.124). Assume the structure disturbance to the time-varying parameters \( u\beta, u_a, u_h \) do not relate to parameters \( \beta, a \) and \( h \) (Nakajima, 2011).

With the increasing popular in using TVP-VAR model to do empirical analysis of monetary policy, more and more researchers concern about variance volatility. Cogley and Sargent (2005) applied stochastic volatility to macroeconomics analysis, also the Primiceri (2005). The specific characteristic of TVP-VAR is allowing parameters volatility with time changing, most of studies that use this model assume the parameters follow the random walk process, not a stationary series process for variance matrix such as AutoRegressive model (AR). While the coefficients are time changing, the variance assumed to be constant, which will cause the estimate parameters to be biased because it ignores the possibility of the volatility variation (Nakajima, 2011). To avoid this misspecification, most studies doing TVP-VAR will consider about stochastic volatility.
in the estimation and analysis. However, stochastic volatility causes the estimation
calculation burden by using classical point estimates like maximum likelihood estimate
due to its limitation. It assume conditional variance is not constant but relies on each of
the time spot in the time series, that means if I have three variables and 124 data, then
maximum likelihood method has to calculate $3^{124}$ times finding every likely value and
comparing to get the value of parameter that maximize likelihood function for the
observed data. Too much calculation makes it impossible for maximum likelihood
method to estimate in my model.

As stated above, the model is in a non-linear form because of the stochastic
volatility, so that we cannot use Durbin and Koopman (2002) modeling which is in linear
Gaussian state space form (Nakajima, 2011). Thus, it needs a specific and more technical
method for sampling. To solve the volatility issue of TVP-VAR model, one feasible
method that Nakajima (2011) used in the paper for sampling to get estimated parameter is
Markov chain Monte Carlo (MCMC) methods which allow a model drawing from
conditional posterior density and it especially works for high dimension estimation.

**MCMC Algorithm**

MCMC combines Markov chain sampling and Monte Carlo integration method to
effectively avoid the problem of function derivation and local extremum convergence
that maximum likelihood estimation has. The most important process in MCMC is using
numerical approximation sampling theory to get the estimated parameters. Markov chain
sampling is constructing a random process from a target distribution which is an
equilibrium probability distribution $\pi$ and treated as the estimated parameter’s posterior
distribution, then use the sample that derived from this posterior distribution $\pi$ to do
Monte Carlo integration. Before doing MCMC, it is important to set the prior probability, because if we can build Markov chain we set an arbitrary start point first and use transition probability in matrix form to iterate Markov chain many times to draw sample. There are several ways to construct a satisfied transition probability, e.g. Gibbs sampler, Metropolis-Hastings algorithm. Above all, MCMC is not an independent sampling, it is an approximately joint sampling distribution under a finite sample.

The estimation procedure Nakajima (2011) used for the TVP-VAR model includes several parts of time varying regression algorithm approach. He derives the posterior distribution using MCMC to get an approximation. Nakajima (2011) first draws sample from the posterior distribution from first 1,000 sample running, then does sampling from posterior distribution set in different sequences and iterate for many times.
Empirical Evidence for U.S. Economy and Monetary Base Growth

Data and Setup

The TVP-VAR model is done in monthly data for the period of 2004 to 2014, which covers three distinct episodes of the Federal Reserve monetary policy: the high target rates during the 2004 to 2006 period with moderate inflation rate; the lower interest rate policy and money lending operations from 2007 to 2009; the quantitative easing period from 2008 to 2014. While monetary policies were differ in each period, they all affect the monetary base as the Fed changes liquidation. The coverage of these periods also prevents the problems in estimation that may result from structural breaks, the problems Inoue and Okimoto (2008) described in the similar Japanese case observed in middle 1990s.

Comparable with the work of Nakajima (2011) and other studies in the literature, I define these three variables: set m as adjusted monetary base\(^1\) in natural logarithm; set y as industrial production index\(^2\) in natural logarithm; set p as inflation rate\(^3\). Thus I have the model (m, y, i) to do TVP-VAR estimation.

I include two lag since lag two has the highest marginal likelihood that I test through one to four lags, according to the marginal likelihood estimation that Nakajima (2009) used to test the different lag lengths, and set intercept as non-time varying coefficient. I used Nakajima(2011) Matlab statistical software package with some modification to run my model.

\(^1\) The adjusted monetary base is taken from Federal Reserve Bank of St. Louis data base. It is in monthly data and seasonally adjusted. http://research.stlouisfed.org/fred2/series/AMBSL

\(^2\) The industrial production index is taken from Federal Reserve Bank of St. Louis data base. It is in monthly data and is seasonally adjusted, Index 2007=100. http://research.stlouisfed.org/fred2/series/INDPRO

\(^3\) The inflation rate is calculated from the Consumer Price Index (CPI) for all urban consumers including all items. The CPI is in monthly data and seasonally adjusted. http://research.stlouisfed.org/fred2/series/CPIAUCSL
MCMC Estimation

Before starting the MCMC simulation, I set the initial value parameters, \( \mu_{\beta 0} = \mu_{a0} = \mu_{h0} = 0 \), and \( \Sigma_{\beta 0} = \Sigma_{a0} = \Sigma_{h0} = 10 \times I \) (Nakajima, 2011), and set default prior specification as following:

\[
(\Sigma_{\beta})^{-2} \sim \text{Gamma} (20, 0.02), (\Sigma_{a})^{-2} \sim \text{Gamma} (4, 0.02), (\Sigma_{h})^{-2} \sim \text{Gamma} (4, 0.02).
\]

I run \( M = 10,000 \) sample to compute the posterior estimates, Table 1 show part of the estimation results.

Table 1

Estimation Results for Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Stdev</th>
<th>95%U</th>
<th>95%L</th>
<th>Geweke</th>
<th>Inef.</th>
</tr>
</thead>
<tbody>
<tr>
<td>sb1</td>
<td>0.0022</td>
<td>0.0003</td>
<td>0.0018</td>
<td>0.0028</td>
<td>0.419</td>
<td>7.96</td>
</tr>
<tr>
<td>sb2</td>
<td>0.0023</td>
<td>0.0003</td>
<td>0.0018</td>
<td>0.0029</td>
<td>0.342</td>
<td>8.17</td>
</tr>
<tr>
<td>sa1</td>
<td>0.0058</td>
<td>0.0017</td>
<td>0.0035</td>
<td>0.0104</td>
<td>0.435</td>
<td>47.71</td>
</tr>
<tr>
<td>sa2</td>
<td>0.0068</td>
<td>0.0024</td>
<td>0.0037</td>
<td>0.0126</td>
<td>0.307</td>
<td>63.23</td>
</tr>
<tr>
<td>sh1</td>
<td>0.6449</td>
<td>0.1409</td>
<td>0.4134</td>
<td>0.9678</td>
<td>0.053</td>
<td>23.15</td>
</tr>
<tr>
<td>sh2</td>
<td>0.4527</td>
<td>0.1333</td>
<td>0.2405</td>
<td>0.7581</td>
<td>0.611</td>
<td>52.56</td>
</tr>
</tbody>
</table>

Note. The sb represents \( \Sigma_{\beta} \), sa represents \( \Sigma_{a} \), sh represents \( \Sigma_{h} \), sb and sa are multiplied by 100. It shows the means, standard deviations, the 95% credible intervals (upper and lower), the Geweke (1992) convergence diagnostics (Geweke) and inefficiency factors (Inef.) for posterior estimates (Nakajima, 2011).

In my result, the estimation has posterior distribution convergence cannot be rejected based on Geweke value, value is in 95% interval range and the inefficiency factors are all small which indicates the two results imply it is an efficient sampling for the parameters estimation.
Figure 2. Estimation results for estimated parameters.

The $s_b$ represents $\Sigma \beta$, $s_a$ represents $\Sigma a$, $s_h$ represents $\Sigma h$, $s_b$ and $s_a$ are already multiplied by 100 in figure 1. The three row of graphs display above from top to bottom are sample auto-correlations, sample paths and posterior densities (Nakajima, 2011).

The sample paths look relatively stable after 10,000 sampling and after the sample autocorrelations first drop they stay steady, which indicate the sampling method works good on generating uncorrelated samples.
Estimation Result for Macroeconomic Dynamic

Figure 3. Graph for variables m, y, p, and their posterior volatility.

In the first row of graphs, m represents monetary base; y represents industrial production index; p represents inflation rate. The second row of graphs show the posterior volatility of these three variables. The solid line is posterior mean, the dash line is one standard deviation band.

From the first row of above graph, monetary base and industrial production has a significant change before and after 2008, while the mean of inflation changing is relatively stable. The second row of above graph shows monetary base and industrial production have significant heteroskedasticity over range of 2007 to 2009, while the variability of inflation is relatively equal across the whole sample period.

Posterior Estimates for Stochastic Volatility and Simultaneous Relation
Figure 4. Stochastic volatility of the structural shock.

This figure shows the posterior estimates of stochastic volatility of m, y, p. The solid line represents posterior mean, the dash line represents ±1 standard deviation bands, and $a_t = \exp(h_t/2)$.

Stochastic volatility of monetary base is relatively stable before 2008 and after that it declined smoothly. Beginning in 2008, the Fed started to significantly expand its balance sheet to increase the money supply after policy rate and discount rate reached historic low. After 2009, the Fed increased the size of the long-term securities purchasing in a smooth pace: from September 2012, the FOMC decided to increase a $40 billion per month in MBS purchasing, with a $45 billion long-term securities purchasing per month which adopted in January 2013, the total purchasing is $85 billion per month right now at a regular pace (monetary policy report, 2014). Thus, the stochastic volatility of monetary base is generally decreasing, the estimated variances of the monetary base shows above can display these monetary policy shocks.
Stochastic volatility of the industrial production experience a massive change after financial crisis full blown in 2008, it sharply declines to very low which is near 0.005 and remains stable from 2010 until now. Stochastic volatility of the inflation to the structure shock is very flat with a slightly decline after 2007, the range is between 0.15 and 0.125 till now.

Figure 5. Posterior estimates for simultaneous relation.

The solid line represents posterior mean, the dash line represents ±1 standard deviation bands. This simultaneous relation of the TVP-VAR model implies the simultaneous effect of one unit of the structural shock to the variables y and p.

The simultaneous relation of the industrial production to the monetary base shock is negative and slightly increase after 2008 period to a positive relation. However, the simultaneous relation of the inflation to monetary base shock is negative and largely increase after 2008 and approach to near zero which indicates the recent condition that the flood of money into the economy does not drive up the inflation at the same, the
annual rate is still 2.0% till 2014. The simultaneous relation of the inflation to the industrial production shock stays negative and relatively flat which shows their opposite relationship and this relationship keeps constant during financial crisis.

**Impulse Response Functions**

The Impulse Response Function (IRF) is a useful method to display the dynamic movements simulated by running VAR model. The IRF gives the $t = s + 1,...,n$ period of response when the system is shocked by a one standard deviation shock. It can describe how the economy reacts over time to exogenous structure shock, e.g. monetary policy shock, productivity shock.

In this model, as the coefficients are time varying, the impulse response is changing over time because it is computed on every time point over the sample period.

*Figure 6.* Impulse responses of TVP-VAR models.
Dotted line represents a one-month lag response to the shock, dashed line represents a half-year lag response to the shock, and solid line represents a one-year lag response to the shock.

The impulse responses function illustrate in figure 6 shows the macroeconomic dynamics effects among the three variables m, y, p. This impulse response is different from the one in VAR whose the shape is somehow like an average movements of the impulse response of the TVP-VAR though. The result in VAR (see figure 1) may not reflect the accurate condition because it only states the average effect of the monetary transmission mechanism to the economy during the whole sample period, but not emphasis the impact of the unconventional monetary policy shock specifically at zero interest rates (Kimura et al., 2003). The different shape may due to the experiences of the 2004 to 2006 higher industry growth contributed to the latter period that smooth out its response after 2007. Therefore, TVP-VAR can better match the macroeconomic variable changing and show the impact of quantitative easing on the U.S. economy.

One standard of monetary base shock significantly affect on industrial production at the beginning in the range of 2007 and 2009. It also shows monetary base shock has much more long-term effect on industrial production rather than short-term effect due to a normally two year policy lag before the industry fully responds to the high growth of monetary base. Therefore, the monetary base transmission mechanism can take effect on output through unconventional monetary policy using direct money injection and long-term securities purchasing.

The reaction of inflation to the one standard deviation monetary base shock shows very moderate movements in the whole period. The trend of this response is close to zero
with slightly up and down movements. This finding is consistent with the current low and stable inflation rate. The impulse response also implies that largely monetary base expanding does not lead to higher inflation within one year lag term. Because one year is still a short term for rigidity wages and price level to make a significant adjustment when monetary base expansion significantly.

The industrial production shock to the inflation is increasing during 2005 to 2007 in one month lag, but decreasing in half-year and one-year lag, and it keeps constant from 2011 till now. The finding indicates that output shock has a diminishing effect on the price level growth. The impact of inflation shock to the monetary base is negative and very stable except a small increase around 2013, which means the monetary base does not response a lot on the influence of inflation. The impact of inflation shock to industrial production is negative in one month period, but the industrial production reacts positively in half-year and one-year lag period, because in a enough time length, the companies recognize the price increasing, then make adjustment on the salary and product price which will in turn boost the price level.
Conclusions

My empirical results suggest that the quantitative monetary easing plus low policy rate and forward guidance which all adopted by the Fed has been achieved the goal that stimulating economic activity when the economy is in zero interest rate condition though the effect is delayed due to policy lag, and well maintaining the inflation target around 2.0%.

The former Federal Reserve Chairman Ben Bernanke said quantitative easing “has helped promote the recovery...asset purchases push down longer-term interest rates and boost asset prices...in turn provided material support to the economy” (Bernanke, 2014). The data from the Fed and U.S. Bureau of Economic\(^4\) show that comparing to 2008 the beginning of financial crisis, the nominal GDP has increased a lot and the unemployment rate has been declined to 6.5%, while inflation has maintained close to the target, prevent the possibility of falling into deflation.

My results also apply to a relatively stable and flat Philips curve. Philips curve is to illustrate a negative relationship between inflation and unemployment. Based on this tradeoff, the Fed can sacrifice inflation to exchange of low unemployment rate through quantitative easing program and direct lending. According to John Keynes’s liquidity preference theory, people had rather keeping cash than doing investment when interest rate is extremely low. After large amount of money inject into the market times money multiplier, interest rate will rise and massive money will bring much more investment opportunities, therefore promote the economy. The Fed Chairman Janet, Yellen believes that the Fed should maintain enough inflation to stimulate business activity.

Though the impulse response function shows a low and stable inflation response to the monetary base shock, it does not reject the possibility that inflation will increase in the long run. If the Fed floods the liquidity into the economy, the large excess money will result in inflation over growth. Since policy has time lag, quantitative easing may not cause high inflation in short run but properly lead to higher inflation in long run. In Milton Friedman’s view, inflation is always and everywhere a monetary phenomenon (Milton Friedman, n. d.). It would not boost output in the long run, nor would output trigger inflation. He against printing money to solve the economic problem as it dilutes the money that people hold in hand. Quantitative easing increases the money amount in circulation, which would depreciate U.S. dollars. Although dollar depreciation makes U.S. export increasing and American companies earn more profit after currency conversion, investors would not like to purchase dollar asset, it becomes harder for government to do further borrowing and payment as dollar is getting lose its value. Considering the unemployment rate is persistent high in a range of 6.5%-7.5% in recent one year, if this high unemployment rate combines with a high inflation, stagflation will come out and hinder the economy revive.

The other risks quantitative easing may carry are: first, the QE program is an extreme policy measure adopted to the severe impaired market and depress economy, thus the implementation and the result are uncertain, and it may lead to global financial market unstable and cause trade deterioration with foreign countries. Second, even with these huge excess reserve, banks and other financial facilities still unwilling to lend it out when the economy has no sign to recover from recession.
References


