ANALYSIS OF POWER SYSTEMS UNDER FAULT CONDITIONS

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ANALYSIS OF POWER SYSTEMS UNDER FAULT CONDITIONS

A Project

by

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Abstract

of

ANALYSIS OF POWER SYSTEM UNDER FAULT CONDITIONS

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The analysis of Power Systems under fault condition represents one of the most important and complex task in Power Engineering. The studies and detection of these faults is necessary to ensure that the reliability and stability of the power system do not suffer a decrement as a result of a critical event such a fault. This project will conduct a research, analyze the behavior of a system under fault conditions and evaluate different scenarios of faults.

____, Committee Chair
Turan Gonen, Ph.D

____
Date

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First of all, I would like to express my gratitude to God for granting me with wisdom and the opportunity of an education.

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Chapter 1

INTRODUCTION

1.1 Overview

During normal operating conditions, current will flow through all elements of the electrical power system within pre-designed values which are appropriate to these elements' ratings. Any power system can be analyzed by calculating the system voltages & currents under normal & abnormal scenarios [2].

Unfortunately, faults could happen as a result of natural events or accidents where the phase will establish a connection with another phase, the ground or both in some cases [4]. A falling tree on a transmission lines could cause a three-phase fault where all phases share a point of contact called fault location. In different occasions, fault could be a result of insulation deterioration, wind damage or human vandalism.

Faults can be defined as the flow of a massive current through an improper path which could cause enormous equipment damage which will lead to interruption of power, personal injury, or death. In addition, the voltage level will alternate which can affect the equipment insulation in case of an increase or could cause a failure of equipment start-up if the voltage is below a minimum level. As a result, the electrical potential difference of the system neutral will increase [2]. Hence, People and equipment will be exposed to the danger of electricity which is not accepted.

In order to prevent such an event, power system fault analysis was introduced. The process of evaluating the system voltages and currents under various types of short circuits is called fault analysis which can determine the necessary safety measures & the required protection system [4]. It is essential to guarantee the safety of public [10]. The analysis of faults leads to appropriate protection settings which can be computed in order to select suitable fuse, circuit breaker size and type of relay [2].

The severity of the fault depends on the short-circuit location, the path taken by fault current, the system impedance and its voltage level. In order to maintain the continuation of power supply to all customers which is the core purpose of the power system existence, all faulted parts must be isolated from the system temporary by the protection schemes. When a fault exists within the relay protection zone at any transmission line, a signal will trip or open the circuit breaker isolating the faulted line. To complete this task successfully, fault analysis has to be conducted in every location assuming several fault conditions. The goal is to determine the optimum protection scheme by determining the fault currents & voltages. In reality, power system can consist of thousands of buses which complicate the task of calculating these parameters without the use of computer software such as Matlab. In 1956, L.W. Coombe and D. G. Lewis proposed the first fault analysis program [4].

There are two types of faults which can occur on any transmission lines; balanced faults & unbalanced faults. In addition, unbalanced faults can be classified into single line-to-ground faults, double line faults and double line-to-ground faults. The most common types taking place in reality are as follow: Line-to-ground fault: this type of fault exists when one phase of any transmission lines establishes a connection with the ground either by ice, wind, falling tree or any other incident. 70% of all transmission lines faults are classified under this category [11].

Line-to-line fault: as a result of high winds, one phase could touch anther phase & line-to-line fault takes place. 15% of all transmission lines faults are considered line-to-line faults [11].

Double line-to-ground: two phases will be involved instead of one at the line-to-ground faults scenarios. 10% of all transmission lines faults are under this type of faults [11].

Three phase fault: in this case, falling tower, failure of equipment or even a line breaking and touching the remaining phases can cause three phase faults. In reality, this type of fault not often exists which can be seen from its share of 5% of all transmission lines faults [11].In order to analyze any unbalanced power system, C.L. Fortescue introduced a method called symmetrical components in 1918 to solve such system using a balanced representation [6]. In this project, literature review section will be provided to summarize the methods used to analyze such cases. Then, a description of the symmetrical components methods will be discussed in detail. Its mathematical model will be presented. After that, a 6-bus system will be under fault for analysis. This analysis will take place using the manual calculations. These results will be compared later with the results of Matlab codes. These results will be discussed in the final chapter and a conclusion will be provided of our comments.

Chapter 2

THE LITERATURE SURVEY

2.1 General

Electric power is generated, transmitted and distributed via large interconnected power systems. The generation of electric power takes place in a power plant. Then the voltage level of the power will be raised by the transformer before the power is transmitted. Electric power is proportional to the product of voltage and current this is the reason why power transmission voltage levels are used in order to minimize power transmission losses [4].

The primary objective of all power systems is to maintain the continuous power supply. During normal operating conditions, current will flow through all elements of the electrical power system within pre-designed values which are appropriate to these elements' ratings. However, natural events such as lightning, weather, ice, wind, heat, failure in related equipment and many other unpredictable factors may lead to undesirable situations and connection between the phases conductors of a transmission lines or the phase conductors to ground, these types of events are known as faults. A falling tree on a transmission lines could cause a three-phase fault where all phases share a point of contact called fault location. In different occasions, fault could be a result of insulation deterioration, wind damage or human vandalism [2, 4].

Faults can be defined as the flow of a massive current through an improper path which could cause enormous equipment damage which will lead to interruption of power, personal injury, or death. In addition, the voltage level will alternate which can affect the equipment insulation in case of an increase or could cause a failure of equipment start-up if the voltage is below a minimum level. As a result, the electrical potential difference of the system neutral will increase. Hence, People and equipment will be exposed to the danger of electricity which is not accepted [2].

Any power system can be analyzed by calculating the system voltages and currents under normal & abnormal scenarios [2].

The fault currents caused by short circuits may be several orders of magnitude larger than the normal operating currents and are determined by the system impedance between the generator voltages and the fault, under the worst scenario if the fault persists, it may lead to long-term power loss, blackouts and permanently damage to the equipment. To prevent such an undesirable situation, the temporary isolation of the fault from the whole system it is necessary as soon as possible. This is accomplished by the protective relaying system [4].

The process of evaluating the system voltages and currents under various types of short-circuits is called fault analysis which can determine the necessary safety measures & the required protection system to guarantee the safety of public [4].

The analysis of faults leads to appropriate protection settings which can be computed in order to select suitable fuse, circuit breaker size and type of relay [2].

The severity of the fault depends on the short-circuit location, the path taken by fault current, the system impedance and its voltage level. In order to maintain the continuation of power supply to all customers which is the core purpose of the power system existence, all faulted parts must be isolated from the system temporary by the protection schemes. When a fault exists within the relay protection zone at any transmission line, a signal will trip or open the circuit breaker isolating the faulted line [4].

To complete this task successfully, fault analysis has to be conducted in every location assuming several fault conditions. The goal is to determine the optimum protection scheme by determining the fault currents & voltages. In reality, power system can consist of thousands of buses which complicate the task of calculating these parameters without the use of computer softwares such as Matlab. In 1956, L.W. Coombe and D. G. Lewis proposed the first fault analysis program [4].

Many exiting texts offer an extensive analysis in fault studies and calculation. Two worth mentioning are Analysis of Faulted Power System by Paul Anderson and Electrical Power Transmission System Engineering Analysis and Design by Turan Gonen. In addition to offer a very illustrative and clear analysis in the fault studies, they also offer an impressive guideline for the power systems analysis understanding in general.

2.2 Type of Faults

There are two types of faults which can occur on any transmission lines; balanced faults and unbalanced faults also known as symmetrical and asymmetrical faults respectively. Most of the faults that occur on power systems are not the balanced three-phase faults, but the unbalances faults. In addition, faults can be categorized as the shunt faults, series faults and simultaneous faults [1]. In the analysis of power system under fault conditions, it is necessary to make a distinction between the types of fault to ensure the best results possible in the analysis. However, for this project only shunt faults are to be analyzed.

2.2.1 Series Faults

Series faults represent open conductor and take place when unbalanced series impedance conditions of the lines are present. Two examples of series fault are when the system holds one or two broken lines, or impedance inserted in one or two lines. In the real world a series faults takes place, for example, when circuit breakers controls the lines and do not open all three phases, in this case, one or two phases of the line may be open while the other/s is closed [1]. Series faults are characterized by increase of voltage and frequency and fall in current in the faulted phases.

2.2.2 Shunt Faults

The shunt faults are the most common type of fault taking place in the field. They involve power conductors or conductor-to-ground or short circuits between conductors. One of the most important characteristics of shunt faults is the increment the current suffers and fall in voltage and frequency. Shunt faults cab be classified into four categories [5].

1. Line-to-ground fault: this type of fault exists when one phase of any transmission lines establishes a connection with the ground either by ice, wind, falling tree or any other incident. 70% of all transmission lines faults are classified under this category [11].

2. Line-to-line fault: as a result of high winds, one phase could touch anther phase & line-to-line fault takes place. 15% of all transmission lines faults are considered line-to-line faults [11].
3. Double line-to-ground: falling tree where two phases become in contact with the ground could lead to this type of fault. In addition, two phases will be involved instead of one at the line-to-ground faults scenarios. 10% of all transmission lines faults are under this type of faults [11].
4. Three phase fault: in this case, falling tower, failure of equipment or even a line breaking and touching the remaining phases can cause three phase faults. In reality, this type of fault not often exists which can be seen from its share of 5% of all transmission lines faults [11].

The first three of these faults are known as asymmetrical faults.

2.3 Method of Analysis

In order to analyze any unbalanced power system, C.L. Fortescue introduced a method called symmetrical components in 1918 to solve such system using a balanced representation [6]. This method is considered the base of all traditional fault analysis approaches of solving unbalanced power systems [4].

The theory suggests that any unbalanced system can be represented by a number of balanced systems equal to the number of its phasors. The balanced systems representations are called symmetrical components. In three-phase system, there are three sets of balanced symmetrical components can be obtain; the positive, negative and zero sequence components. The positive sequence consists of set of phasors which has the same original system sequence. The second set of phasors has an opposite sequence which is called the negative sequence. The zero sequence has three components in phase with each other. The symmetrical components theory will be discuss into more detail in chapter 3 of this project.

Chapter 3

THE MATHEMATICAL MODEL

3.1 Introduction

This chapter describes the mathematical model that is used in the analysis of faulted power systems and the assumptions that are used in this project's analysis.

3.2 Fortescue's Theory

A three-phase balanced fault can be defined as a short circuit with fault impedance called Z_f between the ground and each phase. The short circuit will be called a solid fault when Z_f is equal to zero. This type of fault is considered the most sever short circuit which can affect any electrical system. Fortunately, it is rarely taking place in reality. Fortescue segregated asymmetrical three-phase voltages and currents into three sets of symmetrical components in 1918 [8].

Analyzing any symmetrical fault can be achieved using impedance matrix method or Thevenin's method. Fortescue's theorem suggests that any unbalanced fault can be solved into three independent symmetrical components which differ in the phase sequence. These components consist of a positive sequence, negative sequence and a zero sequence.

3.2.1 Positive Sequence Components

The positive sequence components are equal in magnitude and displayed from each other by 120^o with the same sequence as the original phases. The positive sequence currents and voltages follow the same cycle order of the original source. In the case of typical counter clockwise rotation electrical system, the positive sequence phasor are shown in Fig 3.1. The same case applies for the positive current phasors. This sequence is also called the “abc” sequence and usually denoted by the symbol “+” or “1” [9].

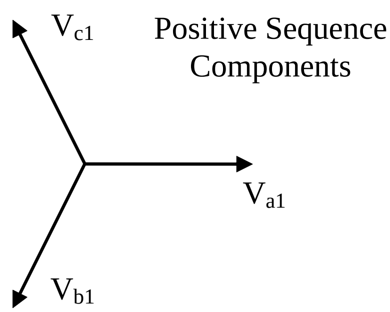

Figure 3.1 Positive sequence components

3.2.2 Negative Sequence Components

This sequence has components that are also equal in magnitude and displayed from each other by 120^o similar to the positive sequence components. However, it has an opposite phase sequence from the original system. The negative sequence is identified as the “acb” sequence and usually denoted by the symbol “-” or “2” [9].The phasors of this sequence are shown in Fig 3.2 where the phasors rotate anti- clockwise. This sequence occurs only in case of an unsymmetrical fault in addition to the positive sequence components,

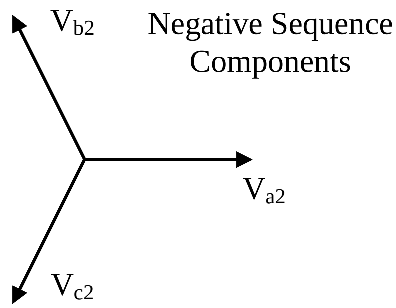

Figure 3.2: Negative sequence components

3.2.3 Zero Sequence Components

In this sequence, its components consist of three phasors which are equal in magnitude as before but with a zero displacement. The phasor components are in phase with each other. This is illustrated in Fig 3.3. Under an asymmetrical fault condition, this sequence symbolizes the residual electricity in the system in terms of voltages and currents where a ground or a fourth wire exists. It happens when ground currents return to the power system through any grounding point in the electrical system. In this type of faults, the positive and the negative components are also present. This sequence is known by the symbol “0” [9].

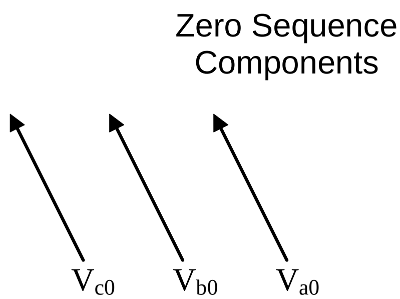

Figure 3.3 Zero sequence components

The following are three sets of components to represent three-phase system voltages as positive, negative and zero components:

Positive V_{a1} V_{b1} V_{c1}

Negative V_{a2} V_{b2} V_{c2}

Zero V_{a0} V_{b0} V_{c0}

The addition of all symmetrical components will present the original system phase components V_a, V_b and V_c as seen below:

$$\begin{aligned} V_a &= V_{a0} + V_{a1} + V_{a2} \\ V_b &= V_{b0} + V_{b1} + V_{b2} \\ V_c &= V_{c0} + V_{c1} + V_{c2} \end{aligned} \tag{3.1}$$

The “a” operator is defined below:

$$a = 1\angle 0^\circ \tag{3.2}$$

The following relations can be driven from 3.2:

$$a^2 = 1\angle -120^\circ$$
$$a^3 = 1\angle 0^\circ$$

From the above definition and using the “a” operator, it can be translated into a set of equations to represents each sequence:

a) Zero sequence components:

$$V_{a0} = V_{b0} = V_{c0} \tag{3.3}$$

b) Positive sequence components:

$$V_{b1} = a^2 V_{a1}$$
$$V_{c1} = a V_{a1} \qquad (3.4)$$

c) Negative sequence components:

$$V_{b2} = a V_{a2}$$
$$V_{c2} = a^2 V_{a2} \qquad (3.5)$$

Now, the original system phasors V_a, V_b and V_c can be expressed in terms of phase “a” components only. Equation 3.1 can be written as follows:

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2} \qquad (3.6)$$
$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}$$

Writing the above equations can be accomplished in a matrix form:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \qquad (3.7)$$

Defining A as:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \qquad (3.8)$$

Equation 3.7 can be written as:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = A \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \qquad (3.9)$$

This equation can be reversed in order to obtain the positive, negative and zero sequences from the system phasors:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = A^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{3.10}$$

Where A^{-1} is equal to the following:

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{3.11}$$

These equations can be applied for the phase voltages and currents. In addition, it can express the line currents and the line-to-line voltages of any power system under fault conditions.

3.3 Sequence Impedances and Sequence Networks

Knowing specific data of the sequence-impedances of synchronous machine, transmission lines and transformers is a need when a numerical analysis of a power system under fault conditions is required.

Due to the extensive information about sequence impedances and sequence network theory a general introduction of the comprehensive concepts of the sequence impedances and sequence networks is here explained [3].

3.3.1 Synchronous Machines

The positive, negative and zero sequence current impedances in synchronous machines and other rotating machines have generally different values. For a synchronous machine the selection of its positive-sequence impedance depends on the time that is assume to elapse from the instant the fault initiates to the instant at which values are desired (e.g., for relay response, breaker opening, or sustained fault conditions), therefore the positive-sequence impedance of a synchronous machine can be selected to be its subtransient ($X_d^{"}$), transient ($X_d^{'}$), or synchronous (X_d) reactance. However, it is the subtransnient reactance of the synchronous machine the one usually taken for fault analysis purposes studies [1].

The subtransient and negative-sequence reactances are the same in the case of a cylindrical-rotor synchronous machine. In general, the negative-sequence impedance of a synchronous machine is usually determined from

$$Z_2 = jX_2 = j\left(\frac{X_d^{"} + X_q^{"}}{2}\right) \tag{3.11}$$

The determination of the zero-sequence impedance of a synchronous machine is a little more complicated since it varies widely and depends on the pitch of the armature coils. The zero-sequence impedance is much smaller than the corresponding positive, negative-sequence impedances. An easy way to measure the zero-sequence impedance is by connecting the three armature windings in series and applying a single-phase voltage [1].

3.3.2 Transmission Line

A transmission line is considered a passive device since no voltage or current sources are present in its equivalent model. It is also considered a bilateral device which means the line behavior remains the same way no matter the direction of the current. It is important to note that although a single transmission line is bilateral, an interconnected transmission network is not this is due to the dispersion of active components (generators) throughout the network.

Due to these characteristics the phase sequence of the applied voltage makes no difference, this means that the a-b-c (positive-sequence) voltages produce the same voltage drops as a-c-b (negative-sequence) voltages. Therefore, the impedances of a transmission line for its positive- and negative-sequence are the same, provided that the line is transposed. A transmission line is transposed when the phase conductors of the line physically exchange positions along the length of the line [7].

Figure 3.4 shows a representation of an untransposed transmission line with unequal self-impedances and unequal mutual impedances [1]

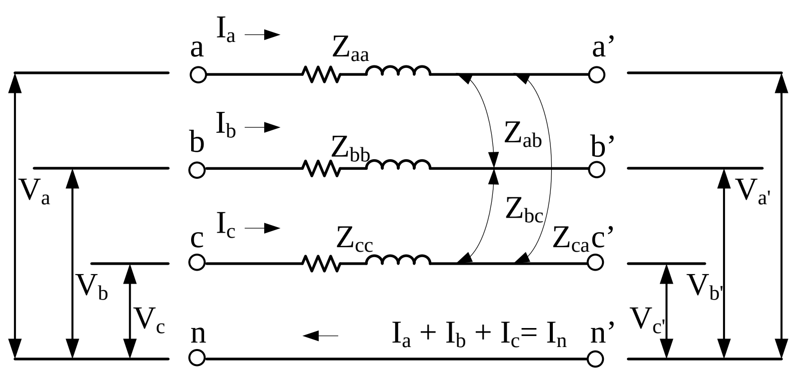

Figure 3.4 Transmission line diagram with unequal impedances

From Figure 3.4 it is observed

$$[V_{abc}] = [Z_{abc}][I_{abc}] \tag{3.12}$$

Where,

$$\left[Z_{abc}\right]=\begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \tag{3.13}$$

In general the self impedances are

$$Z_{aa} \neq Z_{bb} \neq Z_{cc} \tag{3.14}$$

And the mutual impedances

$$Z_{ab} \neq Z_{bc} \neq Z_{ca} \tag{3.15}$$

If both sides of Equation 3.12 are multiplying by $\left[A\right]^{-1}$ and knowing that

$$\left[I_{abc}\right]=\left[A\right]\left[I_{012}\right] \tag{3.16}$$

then,

$$\left[A\right]^{-1}\left[V_{abc}\right]=\left[A\right]^{-1}\left[Z_{abc}\right]\left[A\right]\left[I_{012}\right] \tag{3.17}$$

where,

$$\left[Z_{012}\right] \triangleq \left[A\right]^{-1}\left[Z_{abc}\right]\left[A\right] \tag{3.18}$$

Then sequence impedance matrix of an untransposed transmission line cam be obtained from Equation 3.18 and is expressed as

$$\left[Z_{012}\right]=\begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix} \tag{3.19}$$

it can be also expressed as

$$\left[Z_{012}\right]=\begin{bmatrix} \left(Z_{s0}+2Z_{m0}\right) & \left(Z_{s2}-Z_{m2}\right) & \left(Z_{s1}-Z_{m1}\right) \\ \left(Z_{s1}-Z_{m1}\right) & \left(Z_{s0}-Z_{m0}\right) & \left(Z_{s2}+2Z_{m2}\right) \\ \left(Z_{s2}-Z_{m2}\right) & \left(Z_{s1}-2Z_{m1}\right) & \left(Z_{s0}-Z_{m0}\right) \end{bmatrix} \tag{3.20}$$

it is known by definition that,

$$Z_{s0} = \text{zero-sequence self impedance}$$
$$\triangleq \frac{1}{3}\left(Z_{aa} + Z_{bb} + Z_{cc}\right), \tag{3.21}$$

$$Z_{s1} = \text{positive-sequence self impedance}$$
$$\triangleq \frac{1}{3}\left(Z_{aa} + aZ_{bb} + a^2 Z_{cc}\right), \tag{3.22}$$

$$Z_{s2} = \text{negative-sequence self impedance}$$
$$\triangleq \frac{1}{3}\left(Z_{aa} + a^2 Z_{bb} + aZ_{cc}\right), \tag{3.23}$$

$$Z_{m0} = \text{zero-sequence mutual impedance}$$
$$\triangleq \frac{1}{3}\left(Z_{bc} + Z_{ca} + Z_{ab}\right), \tag{3.24}$$

$$Z_{m1} = \text{positive-sequence mutual impedance}$$
$$\triangleq \frac{1}{3}\left(Z_{bc} + aZ_{ca} + a^2 Z_{ab}\right), \tag{3.25}$$

$$Z_{m2} = \text{negative-sequence mutual impedance}$$
$$\triangleq \frac{1}{3}\left(Z_{bc} + a^2 Z_{ca} + aZ_{ab}\right), \tag{3.26}$$

Therefore,

$$\left[V_{012}\right] = \left[Z_{012}\right]\left[I_{012}\right] \tag{3.27}$$

The resultant matrix from Equation 3.20 is not symmetrical this means that the result obtained from Equation 3.27 will show a non desirable result such as mutual coupling among the three sequence.

There are two solutions to this problem, one is to completely transpose the line and the other one is to obtained equal mutual impedances by placing the conductors with equilateral spacing among them. Figure 3.5 shows a representation of a completely transposed line with equal series and equal mutual impedances. [1]

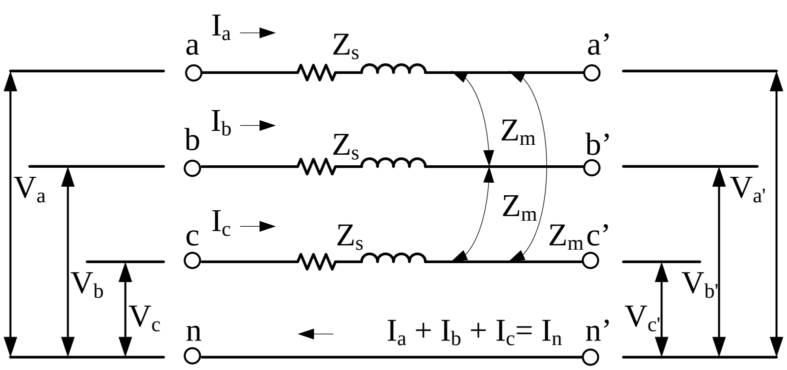

Figure 3.5 Transmission line diagram with equal series and equal mutual impedances

For a transposed line such as the one represented in Figure 3.5 the mutual impedances are

$$Z_{ab} = Z_{bc} = Z_{ca} = Z_m \tag{3.28}$$

And the self-impedances,

$$Z_{aa} = Z_{bb} = Z_{cc} = Z_s \tag{3.29}$$

With Equations 3.28 and 3.29, Equation 3.13 can be expressed as

$$\left[Z_{abc}\right] = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_{21} & Z_s \end{bmatrix} \tag{3.30}$$

Where the self and mutual impedance are given by

$$Z_s = \left[\left(r_a + r_e\right) + j0.1213\ln\frac{D_e}{D_s}\right] l\Omega, \tag{3.31}$$

$$Z_m = \left[r_e + j0.1213 \ln \frac{D_e}{D_{eq}} \right] l\Omega, \tag{3.32}$$

Where r_e is the resistance of Carson's, D_s is the geometric mean radius (GMR) and D_e is a function of both the earth resistivity and the frequency.

And,

$$D_{eq} \approx D_m = \left(D_{ab} \times D_{bc} \times D_{ca} \right)^{1/3}, \tag{3.33}$$

When Equation 3.18 is applied

$$\left[Z_{012} \right] = \begin{bmatrix} \left(Z_s + 2Z_m \right) & 0 & 0 \\ 0 & \left(Z_s - Z_m \right) & 0 \\ 0 & 0 & \left(Z_s - Z_m \right) \end{bmatrix}, \tag{3.34}$$

Equation 3.34 can also be expressed as

$$\left[Z_{012} \right] = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}, \tag{3.35}$$

As it can be seen from equations 3.34 and 3.35, the result is desirable since there is no mutual coupling among the three sequences. This means that each sequence network only produce voltage drop in their respective sequence network. [1]

3.3.3 Transformers

Three identical single-phase transformers can be connected to make up a three-phase transformer. This is known as a three-phase transform bank. The positive and negative sequence currents in a transformer are the same. However, the zero-sequence series impedances of three-units are a little different than the positive-, and negative-sequence series impedance, but in practice all the sequences impedances are assumed to be the same regardless the transformer type.

$$Z_0 = Z_1 = Z_2 = Z_{trf} \qquad (3.36)$$

Z_0 is infinite when the flow of zero-sequence current is prevented by the transformer connection.

The zero-sequence networks diagrams of three phase transformer banks are shown in Figure 3.6. If the zero sequence current path is not indicated in the diagram this means the transformer connection prevents the flow of it.

Some points are important to highlight, zero sequence current flows inside the delta windings for a delta-delta bank but prevents it to flow outside the windings by not providing a path for it. [1]

Also, it is important to notice that there is no existing path that allows the flow of the zero-sequence current in a wye-grounded-wye-connected three-phase transformer bank. The reason is no zero-sequence current is present in any given winding on the wye side of the transformer bank since it has an ungrounded wye connection. [1]

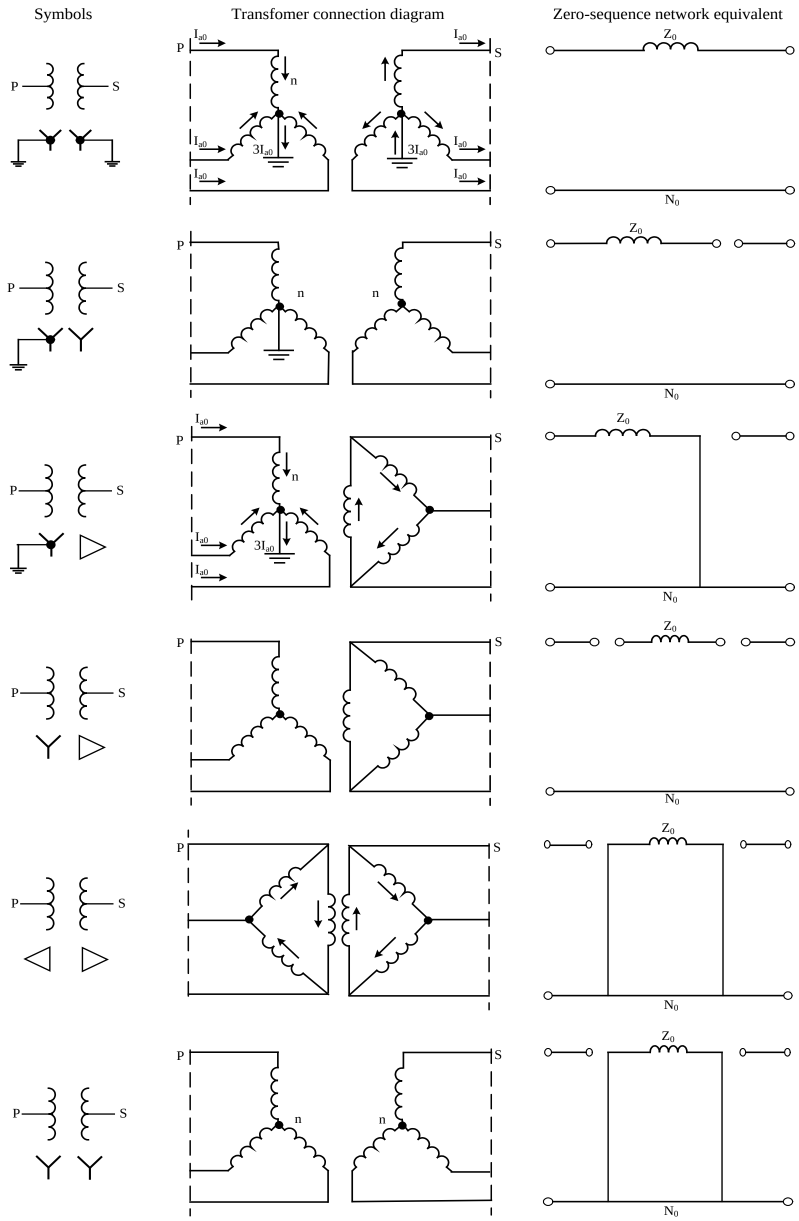

Figure 3.6 Zero-sequence network equivalents of three-phase banks

The equivalent networks for the zero-sequence of three-phase transformer banks made of three indentical single-phase transformers with three windings are shown in figure 3.7. It is important to note that for a three-phase transformer bank made with three identical single-phase transformers with three windings the wye-wye connection with delta tertiary is the only one that allows the flow of zero-sequence current from either wye line (as long as the are grounded). [1]

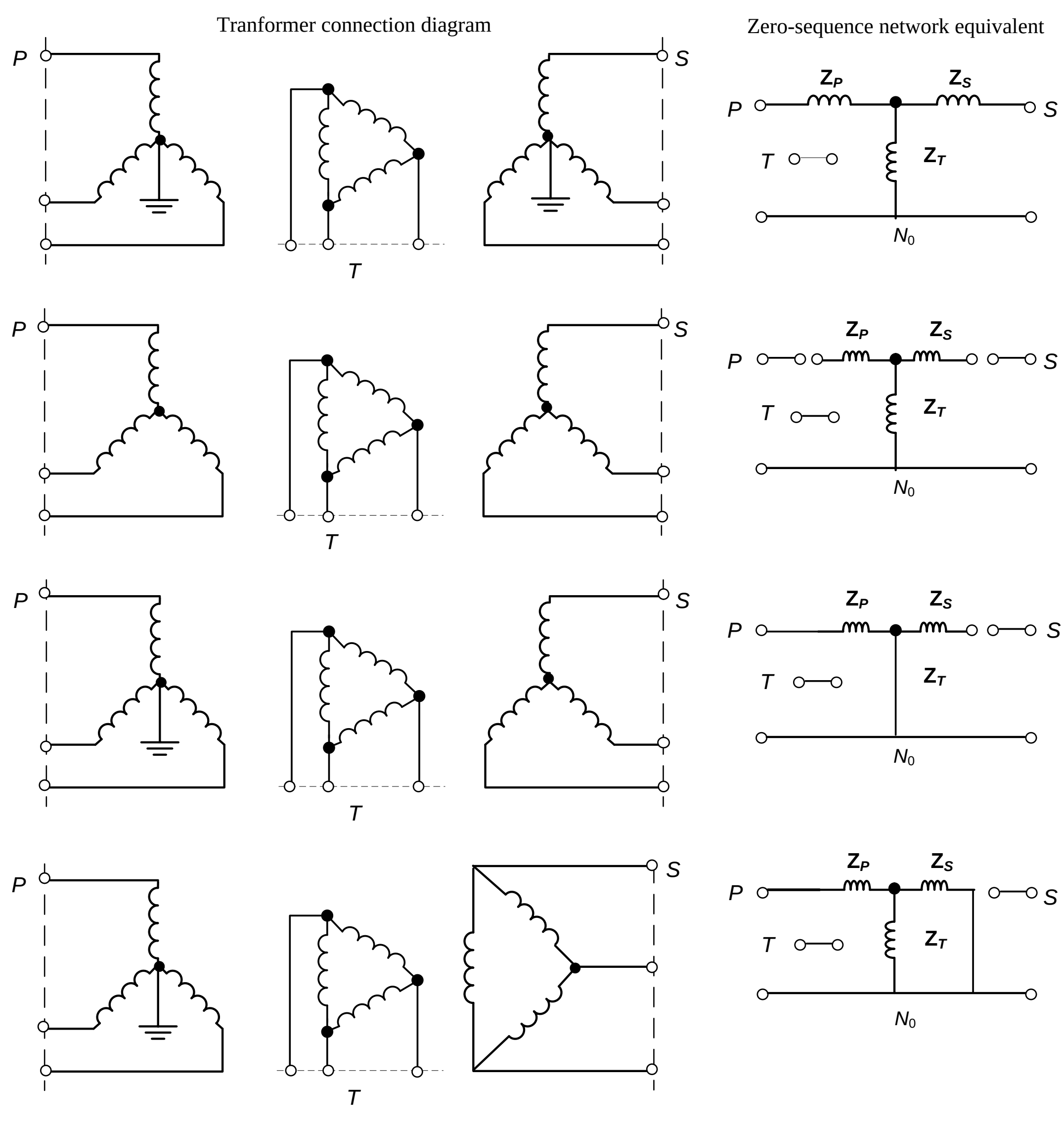

Figure 3.7 Zero-sequence network equivalents of three windings transformer banks

3.4 Fault Analysis in Power Systems

In general, a fault is any event, unbalanced situation or any asymmetrical situation that interferes with the normal current flow in a power system and forces voltages and currents to differ from each other.

It is important to distinguish between series and shunt faults in order to make an accurate fault analysis of an asymmetrical three-phase system. When the fault is caused by an unbalance in the line impedance and does not involve a ground, or any type of inter-connection between phase conductors it is known as a series fault. On the other hand, when the fault occurs and there is an inter-connection between phase-conductors or between conductor(s) and ground and/or neutral it is known as a shunt fault. [3]

Statistically, series faults do not occur as often as shunt faults does. Because of this fact only the shunt faults are explained here in detail since the emphasis in this project is on analysis of a power system under shunt faults.

3.4.1 Three-Phase Fault

By definition a three-phase fault is a symmetrical fault. Even though it is the least frequent fault, it is the most dangerous. Some of the characteristics of a three-phase fault are a very large fault current and usually a voltage level equals to zero at the site where the fault takes place. [3]

A general representation of a balanced three-phase fault is shown in Figure 3.8 where F is the fault point with impedances Z_f and Z_g . Figure 3.9 shows the sequences networks interconnection diagram.

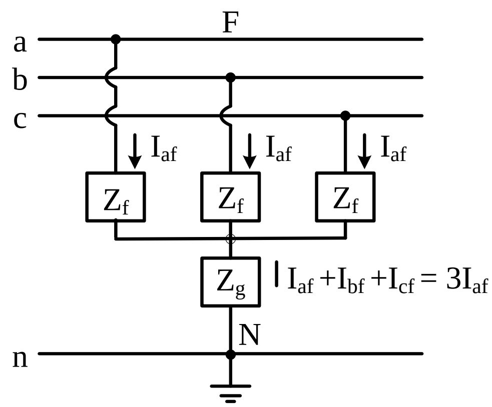

Figure 3.8 General representation of a balanced three-phase fault

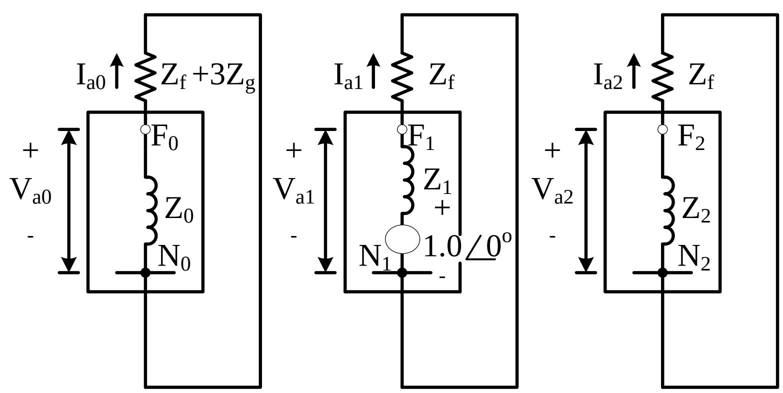

Figure 3.9 Sequence network diagram of a balanced three-phase fault

From Figure 3.9 it can be noticed that the only one that has an internal voltage source is the positive-sequence network. Therefore, the corresponding currents for each of the sequences can be expressed as

$$
\begin{aligned}
I_{a0} &= 0 \\
I_{a2} &= 0 \\
I_{a1} &= \frac{1.0\angle 0^\circ}{Z_1 + Z_f}
\end{aligned}
\tag{3.37}
$$

If the fault impedance Z_f is zero,

$$I_{a1} = \frac{1.0\angle 0°}{Z_1} \qquad (3.38)$$

If equation is substituted into equation

$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{a1} \\ 0 \end{bmatrix} \qquad (3.39)$$

Solving Equation 3.39

$$\begin{aligned} I_{af} &= I_{a1} = \frac{1.0\angle 0°}{Z_1 + Z_f}, \\ I_{bf} &= a^2 I_{a1} = \frac{1.0\angle 240°}{Z_1 + Z_f}, \\ I_{cf} &= a I_{a1} = \frac{1.0\angle 120°}{Z_1 + Z_f} \end{aligned} \qquad (3.40)$$

Since the sequence networks are short-circuited over their own fault impedance

$$\begin{aligned} V_{a0} &= 0 \\ V_{a1} &= Z_f I_{a1} \\ V_{a2} &= 0 \end{aligned} \qquad (3.41)$$

If Equation is substituted into Equation

$$\begin{bmatrix} V_{af} \\ V_{bf} \\ V_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ V_{a1} \\ 0 \end{bmatrix} \qquad (3.42)$$

Therefore,

$$\begin{aligned} V_{af} &= V_{a1} = Z_f I_{a1} \\ V_{bf} &= a^2 V_{a1} = Z_f I_{a1} \angle 240^\circ \\ V_{cf} &= a V_{a1} = Z_f I_{a1} \angle 120^\circ \end{aligned} \tag{3.43}$$

The line-to-line voltages are

$$\begin{aligned} V_{ab} &= V_{af} - V_{bf} = V_{a1}\left(1 - a^2\right) = \sqrt{3} Z_f I_{a1} \angle 30^\circ \\ V_{bc} &= V_{bf} - V_{cf} = V_{a1}\left(a^2 - a\right) = \sqrt{3} Z_f I_{a1} \angle -90^\circ \\ V_{ca} &= V_{cf} - V_{af} = V_{a1}\left(a - 1\right) = \sqrt{3} Z_f I_{a1} \angle 150^\circ \end{aligned} \tag{3.44}$$

If Z_f equals to zero,

$$\begin{aligned} I_{af} &= \frac{1.0\angle 0^\circ}{Z_1} \\ I_{bf} &= \frac{1.0\angle 240^\circ}{Z_1}, \\ I_{cf} &= \frac{1.0\angle 120^\circ}{Z_1} \end{aligned} \tag{3.45}$$

The phase voltages becomes,

$$\begin{aligned} V_{af} &= 0 \\ V_{bf} &= 0 \\ V_{cf} &= 0 \end{aligned} \tag{3.46}$$

And the line voltages,

$$\begin{aligned} V_{a0} &= 0 \\ V_{a1} &= 0 \\ V_{a2} &= 0 \end{aligned} \tag{3.47}$$

3.4.2 Single Line-to-Ground Fault

The single line-to-ground fault is usually referred as "short circuit" fault and occurs when one conductor falls to ground or makes contact with the neutral wire. The general representation of a single line-to-ground fault is shown in Figure 3.10 where F is the fault point with impedances Z_f. Figure 3.11 shows the sequences network diagram. Phase a is usually assumed to be the faulted phase, this is for simplicity in the fault analysis calculations. [1]

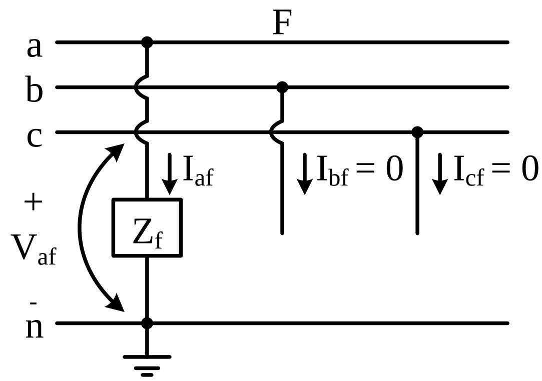

Figure 3.10 General representation of a single line-to-ground fault.

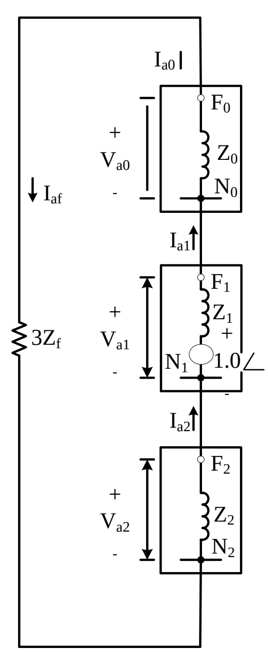

Figure 3.11 Sequence network diagram of a single line-to-ground fault.

Since the zero-, positive-, and negative-sequence currents are equals as it can be observed in Figure 3.11. Therefore,

$$I_{a0} = I_{a1} = I_{a2} = \frac{1.0\angle 0^\circ}{Z_0 + Z_1 + Z_2 + 3Z_f} \tag{3.48}$$

Since

$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{3.49}$$

Solving Equation the fault current for phase a is

$$I_{af} = I_{a0} + I_{a1} + I_{a2} \tag{3.50}$$

it can also be

$$I_{af} = 3I_{a0} = 3I_{a1} = 3I_{a2} \tag{3.51}$$

From Figure 3.10 it can be observed that,

$$V_{af} = Z_f I_{af} \tag{3.52}$$

The voltage at faulted phase a can be obtained by substituting Equation 3.49 into Equation 3.52. Therefore,

$$V_{af} = 3Z_f I_{a1} \tag{3.53}$$

but,

$$V_{af} = V_{a0} + V_{a1} + V_{a2} \tag{3.54}$$

therefore,

$$V_{a0} + V_{a1} + V_{a2} = 3Z_f I_{a1} \tag{3.55}$$

With the results obtained for sequence currents, the sequence voltages can be obtained from

$$\begin{bmatrix} V_{a0} \\ V_{b1} \\ V_{c2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0\angle 0^\circ \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{3.56}$$

By solving Equation

$$\begin{aligned} V_{a0} &= -Z_0 I_{a0} \\ V_{a1} &= 1.0 - Z_1 I_{a1} \\ V_{a2} &= -Z_2 I_{a2} \end{aligned} \tag{3.57}$$

If the single line-to-ground fault occurs on phase b or c, the voltages can be found by the relation that exists to the known phase voltage components,

$$\begin{bmatrix} V_{af} \\ V_{bf} \\ V_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \tag{3.58}$$

as

$$\begin{aligned} V_{bf} &= V_{a0} + a^2 V_{a1} + a V_{a2} \\ V_{cf} &= V_{a0} + a V_{a1} + a^2 V_{a2} \end{aligned} \tag{3.59}$$

3.4.3 Line-to-Line Fault

A line-to-line fault may take place either on an overhead and/or underground transmission system and occurs when two conductors are short-circuited. One of the characteristic of this type of fault is that its fault impedance magnitude could vary over a wide range making very hard to predict its upper and lower limits. It is when the fault impedance is zero that the highest asymmetry at the line-to-line fault occurs [3].

The general representation of a line-to-line fault is shown in Figure 3.12 where F is the fault point with impedances Z_f. Figure 3.13 shows the sequences network diagram. Phase b and c are usually assumed to be the faulted phases; this is for simplicity in the fault analysis calculations [1],

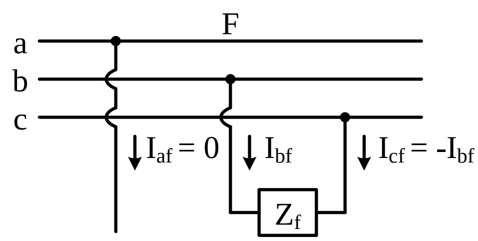

Figure 3.12 Sequence network diagram of a line-to-line fault.

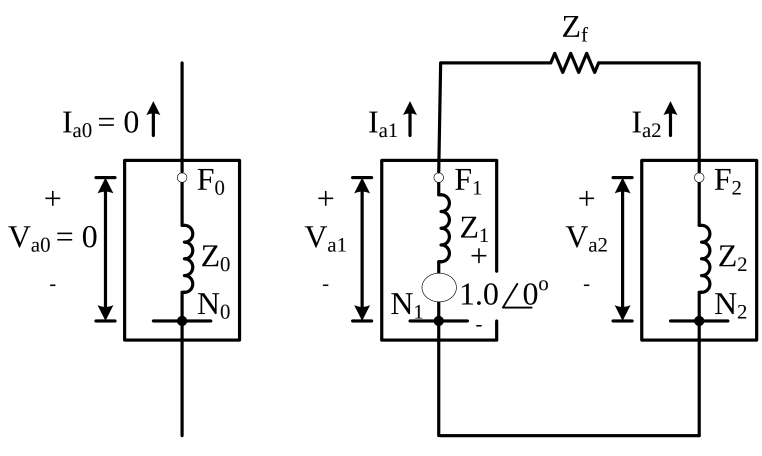

Figure 3.13 Sequence network diagram of a single line-to-ground fault.

From Figure 3.13 it can be noticed that

$$\begin{aligned} I_{af} &= 0 \\ I_{bf} &= -I_{cf} \\ V_{bc} &= Z_f I_{bf} \end{aligned} \tag{3.60}$$

And the sequence currents can be obtained as

$$I_{a0} = 0 \tag{3.61}$$

$$I_{a1} = -I_{a2} = \frac{1.0\angle 0^\circ}{Z_1 + Z_2 + Z_f} \tag{3.62}$$

If $Z_f = 0$,

$$I_{a1} = -I_{a2} = \frac{1.0\angle 0^\circ}{Z_1 + Z_2} \tag{3.63}$$

The fault currents for phase b and c can be obtained by substituting Equations 3.61 and 3.62 into Equation 3.49

$$I_{bf} = -I_{cf} = \sqrt{3}\ I_{a1}\angle -90^\circ \tag{3.64}$$

The sequence voltages can be found similarly by substituting Equations 3.61 and 3,62 into Equation 3.56

$$\begin{aligned} V_{a0} &= 0 \\ V_{a1} &= 1.0 - Z_1\ I_{a1} \\ V_{a2} &= -Z_2\ I_{a2} = Z_2\ I_{a1} \end{aligned} \tag{3.65}$$

Also substituting Equation 3.65 into Equation 3.58

$$\begin{aligned} V_{af} &= V_{a1} + V_{a2} = 1.0 + I_{a1}(Z_2 - Z_1) \\ V_{bf} &= a^2 V_{a1} + a V_{a2} = a^2 + I_{a1}(aZ_2 - a^2 Z_1) \\ V_{cf} &= a V_{a1} + a^2 V_{a2} = a + I_{a1}(a^2 Z_2 - aZ_1) \end{aligned} \tag{3.66}$$

Finally, the line-to-line voltages for a line-to-line fault can be expressed as

$$\begin{aligned} V_{ab} &= V_{af} - V_{bf} \\ V_{bc} &= V_{bf} - V_{cf} \\ V_{ca} &= V_{cf} - V_{af} \end{aligned} \tag{3.67}$$

3.4.4 Double Line-to-Ground Fault

A double line-to-ground fault represents a serious event that causes a significant asymmetry in a three-phase symmetrical system and it may spread into a three-phase fault when not clear in appropriate time. The major problem when analyzing this type of fault is the assumption of the fault impedance Z_f , and the value of the impedance towards the ground $Z_{g.}$ [3]

The general representation of a double line-to-ground fault is shown in Figure 3.14 where F is the fault point with impedances Z_f and the impedance from line to ground Z_g . Figure 3.15 shows the sequences network diagram. Phase b and c are assumed to be the faulted phases, this is for simplicity in the fault analysis calculations. [1]

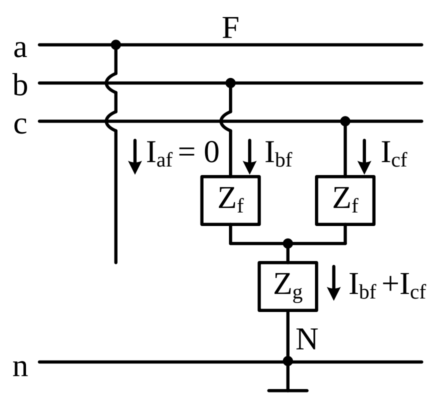

Figure 3.14 General representation of a double line-to-ground fault.

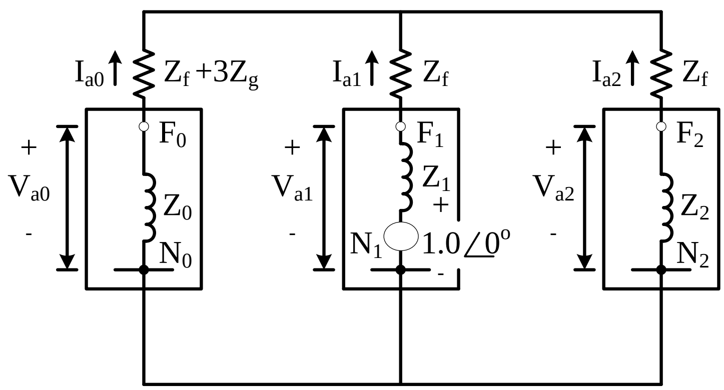

Figure 3.15 Sequence network diagram of a double line-to-ground fault.

From Figure 3.15 it can be observed that

$$\begin{aligned} I_{af} &= 0 \\ V_{bf} &= (Z_f + Z_g)I_{bf} + Z_g I_{cf} \\ V_{cf} &= (Z_f + Z_g)I_{cf} + Z_g I_{bf} \end{aligned} \tag{3.68}$$

Based on Figure 3.15, the positive-sequence currents can be found as

$$I_{a1} = \frac{1.0\angle 0^\circ}{(Z_1 + Z_f) + \frac{(Z_2 + Z_f)(Z_0 + Z_f + 3Z_g)}{(Z_2 + Z_f) + (Z_0 + Z_f + 3Z_g)}}$$

$$I_{a2} = -\left[\frac{(Z_0 + Z_f + 3Z_g)}{(Z_2 + Z_f) + (Z_0 + Z_f + 3Z_g)}\right] I_{a1} \tag{3.69}$$

$$I_{a0} = -\left[\frac{(Z_2 + Z_f)}{(Z_2 + Z_f) + (Z_0 + Z_f + 3Z_g)}\right] I_{a1}$$

An alternative method is,

$$\begin{aligned} I_{af} &= 0 = I_{a0} + I_{a1} + I_{a2} \\ I_{a0} &= -(I_{a1} + I_{a2}) \end{aligned} \tag{3.70}$$

If Z_f and Z_g are both equal to zero, then the positive-, negative-, and zero-sequences can be obtained from

$$I_{a1} = \frac{1.0\angle 0^{\circ}}{(Z_1) + \frac{(Z_2)(Z_0)}{(Z_2 + Z_0)}}$$

$$I_{a2} = -[\frac{(Z_0)}{(Z_2 + Z_0)}]I_{a1} \tag{3.71}$$

$$I_{a0} = -[\frac{(Z_2)}{(Z_2 + Z_0)}]I_{a1}$$

From Figure 3.14 the current for phase a is

$$I_{af} = 0 \tag{3.72}$$

Now, substituting Equations 3.71 into Equation 3.49 to obtain phase b and c fault currents

$$\begin{aligned} I_{bf} &= I_{a0} + a^2 I_{a1} + aI_{a2} \\ I_{cf} &= I_{a0} + aI_{a1} + a^2 I_{a2} \end{aligned} \tag{3.73}$$

The total fault current flowing into the neutral is

$$I_n = 3I_{a0} = I_{bf} + I_{cf} \tag{3.74}$$

And the sequences voltages can be obtained by using Equation 3.51

$$\begin{aligned} V_{0a} &= -Z_0 I_{a0} \\ V_{a1} &= 1.0 - Z_1 I_{a1} \\ V_{a2} &= -Z_2 I_{a2} \end{aligned} \tag{3.75}$$

The phase voltages are equal to

$$\begin{aligned} V_{af} &= V_{a0} + V_{a1} + V_{a2} \\ V_{bf} &= V_{a0} + a^2 V_{a1} + aV_{a2} \\ V_{cf} &= V_{a0} + aV_{a1} + a^2 V_{a2} \end{aligned} \tag{3.76}$$

The line-to-line voltages can be obtained from

$$V_{ab} = V_{af} - V_{bf}$$
$$V_{bc} = V_{bf} - V_{cf}$$
$$V_{ca} = V_{cf} - V_{af} \quad (3.77)$$

If $Z_f = 0$ and $Z_g = 0$ then the sequence voltages become, and the positive-sequence current is found by using Equation 3.71.

$$V_{a0} = V_{a1} = V_{a2} = 1.0 - Z_1 I_{a1} \quad (3.78)$$

Now the negative- and zero-sequence currents can be obtained from

$$I_{a2} = -\frac{V_{a2}}{Z_2}$$
$$I_{a0} = -\frac{V_{a0}}{Z_0} \quad (3.79)$$

The resultant phase voltages from the relationship given in Equation 3.78 can be expressed as

$$V_{af} = V_{a0} + V_{a1} + V_{a2} = 3V_{a1}$$
$$V_{bf} = V_{cf} = 0 \quad (3.80)$$

And the line-to-line voltages are

$$V_{abf} = V_{af} - V_{bf} = V_{af}$$
$$V_{bcf} = V_{bf} - V_{cf} = 0$$
$$V_{caf} = V_{cf} - V_{af} = -V_{af} \quad (3.81)$$

Chapter 4

APPLICATION OF MATHEMATICAL MODEL

In order to do the analysis a 6-node network well-known circuit introduced by Ward and Hale is used in the following section [2]. The mathematical model previously explained in chapter 3 is applied by doing hand calculations as well as a Matlab code to confirm the results obtained by hand. Figure 4.1 shows the 6-network system. The parameters of the system are given in Appendix A and were taken from Faulted Power System by Paul M. Anderson.

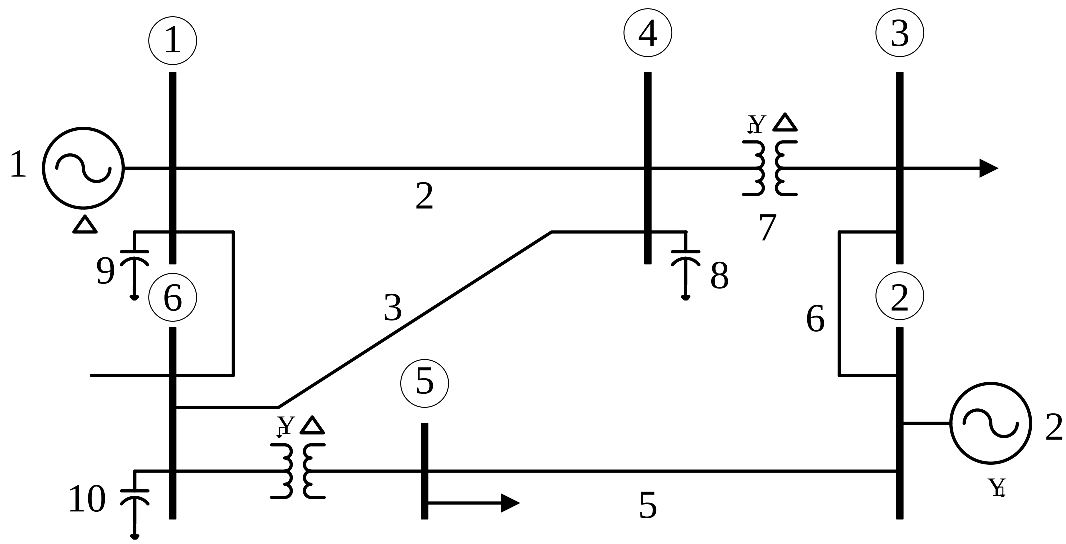

Figure 4.1 Ward and Hale 6-node network [4].

4.1 Hand Calculations

Hand calculation was done based on the mathematical model and Equations explained in Chapter 3 and shows the computations for total fault current, buses voltages, fault currents for each line, sequence currents and phase currents for each shunt fault at bus 4.

4.1.1 Three-Phase Fault

The total fault current at bus 4 for a three-phase fault is given by

$$I_{k(F)} = \frac{V_{k(0)}}{(Z_{kk}^{1} + Z_f)} \tag{4.1}$$

where k is the number of the bus where the fault takes place,

$$\begin{aligned} I_{4(F)} &= \frac{V_{4(0)}}{(Z_{44}^{1} + Z_f)} \\ &= \frac{1\angle 0^\circ}{[(0.13269 + j0.57694) + 0]} \\ &= 1.689183\angle -77.04782^\circ \end{aligned} \tag{4.2}$$

Voltages at all buses

$$V_{i(F)} = V_{i(0)} - Z_{ik} I_k = V_{i(0)} - Z_{i4} I_4 \tag{4.3}$$

For buses 1, 3, 4 and 6, then i = 1,3,4,6 their voltages respectively are

$$\begin{aligned} V_{1(F)} &= V_{1(0)} - Z_{14} I_4 \\ &= 1.05\angle 0^\circ - (0.02254 + j0.17266)(1.689183\angle -77.04782^\circ) \\ &= 0.75776\angle -2.137^\circ \end{aligned} \tag{4.4}$$

$$\begin{aligned} V_{3(F)} &= V_{3(0)} - Z_{34} I_4 \\ &= 1\angle 0^\circ - (0.14333 + j0.53368)(1.689183\angle -77.04782^\circ) \\ &= 0.07525\angle 26.769^\circ \end{aligned} \tag{4.5}$$

$$\begin{aligned} V_{4(F)} &= V_{4(0)} - Z_{44} I_4 \\ &= 1\angle 0^\circ - (0.13269 + j0.57694)(1.689183\angle -77.04782^\circ) \\ &= 0\angle 0^\circ \end{aligned} \tag{4.6}$$

$$\begin{aligned} V_{6(F)} &= V_{6(0)} - Z_{64} I_4 \\ &= 1\angle 0^\circ - (0.06881 + j0.34726)(1.689183\angle -77.04782^\circ) \\ &= 0.40269\angle -2.59^\circ \end{aligned} \tag{4.7}$$

the line fault currents are given by

$$I_{l4(F)} = \frac{V_{l(F)} - V_{4(F)}}{(Z_{l4})_{\text{Actual}}} = \frac{V_{l(F)} - 0}{(Z_{l4})_{\text{Actual}}} = \frac{V_{l(F)}}{(Z_{l4})_{\text{Actual}}} \tag{4.8}$$

For line between bus 1 and bus 4 and applying Equation (4.8) with l = 1

$$\begin{aligned} I_{14(F)} &= \frac{V_{1(F)}}{(Z_{14})_{\text{Actual}}} \\ &= \frac{0.75776\angle -2.137}{(0.16 + j0.74)} = 1\angle -79.936^\circ \end{aligned} \tag{4.9}$$

For line between bus 3 and bus 4 and applying Equation (4.8) with l = 3

$$\begin{aligned} I_{34(F)} &= \frac{V_{3(F)}}{(Z_{34})_{\text{Actual}}} \\ &= \frac{0.07525\angle 26.769^\circ}{(0 + j0.26)} = 0.2894\angle -63.231^\circ \end{aligned} \tag{4.10}$$

For line between bus 6 and bus 4 and applying Equation (4.8) l = 4

$$I_{64(F)} = \frac{V_{6(F)}}{(Z_{64})_{\text{Actual}}}$$

$$= \frac{0.40269\angle -2.59^\circ}{(0.194 + j0.814)} = 0.4812\angle -79.1848^\circ \qquad (4.11)$$

4.1.2 Single Line-to-Ground Fault

The voltage at bus 4 before the fault is

$$V_{4(0)} = 1\angle 0^\circ \qquad (4.12)$$

The sequence currents at bus 4 for a single line-to-ground fault are given by

$$I_4^0 = I_4^1 = I_4^2 \quad \frac{V_{4(0)}}{(Z_{44}^0 + Z_{44}^1 + Z_{44}^2 + 3Z_f)}$$

$$= \frac{1\angle 0^\circ}{(0.00756 + j0.24138) + 2(0.13269 + j0.57694) + 0} \qquad (4.13)$$

$$= 0.70338\angle -78.931^\circ$$

From Equation 3.49

$$I_4^{abc}{}_{(F)} = AI_4^{012} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_4^0 \\ I_4^1 \\ I_4^2 \end{pmatrix} \qquad (4.14)$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \end{pmatrix} \begin{pmatrix} 0.70338\angle -78.931^\circ \\ 0.70338\angle -78.931^\circ \\ 0.70338\angle -78.931^\circ \end{pmatrix}$$

$$= \begin{pmatrix} 2.1101\angle -78.931^\circ \\ 0^\circ \\ 0^\circ \end{pmatrix}$$

The phase currents are

$$I_{4(F)}^{a} = 2.1101\angle -78.931^{\circ}$$
$$I_{4(F)}^{b} = I_{4(F)}^{c} = 0 \tag{4.15}$$

The total fault current at bus 4 is

$$I_{4(F)} = 3I_4^0 = 3(0.70338\angle -78.931^{\circ})$$
$$= 2.1101\angle -78.931^{\circ} \tag{4.16}$$

From Equation 3.57 the sequence voltages are

$$V_{i(F)}^{0} = 0 - Z_{i4}^{0} I_4^0$$
$$V_{i(F)}^{1} = V_{i(0)} - Z_{i4}^{1} I_4^1 \tag{4.17}$$
$$V_{i(F)}^{2} = 0 - Z_{i4}^{2} I_4^2$$

For Bus 1 then i = 1

$$V_{1(F)}^{0} = 0 - Z_{14}^{0} I_4^0$$
$$= 0 - (-0.0112 + j0.11474)(0.70338\angle -78.931^{\circ}) \tag{4.18}$$
$$= 0.08108\angle -163.355^{\circ}$$

$$V_{1(F)}^{1} = V_{1(0)} - Z_{14}^{1} I_4^1$$
$$= 1.05\angle 0^{\circ} - (0.02254 + j0.17266)(0.70338\angle -78.931^{\circ}) \tag{4.19}$$
$$= 0.9278\angle -0.479^{\circ}$$

$$V_{1(F)}^{2} = 0 - Z_{14}^{2} I_4^2$$
$$= 0 - (0.02254 + j0.17266)(0.70338\angle -78.931^{\circ}) \tag{4.20}$$
$$= 0.1224\angle -176.368^{\circ}$$

For Bus 3 then i = 3

$$V_{3(F)}^{0} = 0 - Z_{34}^{0} I_4^0$$
$$= 0 - (0)(0.70338\angle -78.931^{\circ}) \tag{4.21}$$
$$= 0\angle 0^{\circ}$$

$$
\begin{aligned}
V_{3\,(F)}^{1} &= V_{3\,(0)} - Z_{34}^{1} I_{4}^{1} \\
&= 1\angle 0^\circ - (0.14333 + j0.53368)(0.70338\angle -78.931^\circ) \\
&= 0.6128\angle 2.513^\circ
\end{aligned}
\tag{4.22}
$$

$$
\begin{aligned}
V_{3\,(F)}^{2} &= 0 - Z_{34}^{2} I_{4}^{2} \\
&= 0 - (0.14333 + j0.53368)(0.70338\angle -78.931^\circ) \\
&= 0.3889\angle 176.035^\circ
\end{aligned}
\tag{4.23}
$$

For Bus 4 then i = 4

$$
\begin{aligned}
V_{4\,(F)}^{0} &= 0 - Z_{44}^{0} I_{4}^{0} \\
&= 0 - (0.00756 + j0.24138)(0.70338\angle -78.931^\circ) \\
&= 0.1698\angle -170.72^\circ
\end{aligned}
\tag{4.24}
$$

$$
\begin{aligned}
V_{4\,(F)}^{1} &= V_{4\,(0)} - Z_{44}^{1} I_{4}^{1} \\
&= 1\angle 0^\circ - (0.13269 + j0.57694)(0.70338\angle -78.931^\circ) \\
&= 0.5839\angle 1.3426^\circ
\end{aligned}
\tag{4.25}
$$

$$
\begin{aligned}
V_{4\,(F)}^{2} &= 0 - Z_{44}^{2} I_{4}^{2} \\
&= 0 - (0.13269 + j0.57694)(0.70338\angle -78.931^\circ) \\
&= 0.416\angle 178.11^\circ
\end{aligned}
\tag{4.26}
$$

For Bus 6 then i = 6

$$
\begin{aligned}
V_{6\,(F)}^{0} &= 0 - Z_{64}^{0} I_{4}^{0} \\
&= 0 - (-0.01706 + j0.05554)(0.70338\angle -78.931^\circ) \\
&= 0.0408\angle -151.85^\circ
\end{aligned}
\tag{4.27}
$$

$$
\begin{aligned}
V_{6\,(F)}^{1} &= V_{6\,(0)} - Z_{64}^{1} I_{4}^{1} \\
&= 1\angle 0^\circ - (0.06881 + j0.34726)(0.70338\angle -78.931^\circ) \\
&= 0.7509\angle 0.0461^\circ
\end{aligned}
\tag{4.28}
$$

$$
\begin{aligned}
V_{6\,(F)}^{2} &= 0 - Z_{64}^{2} I_{4}^{2} \\
&= 0 - (0.06881 + j0.34726)(0.70338\angle -78.931^\circ) \\
&= 0.249\angle 179.86^\circ
\end{aligned}
\tag{4.29}
$$

the line fault currents are given by

$$I^0_{l4(F)} = \frac{V^0_{l}(F) - V^0_{4}(F)}{(Z^0_{l4})_{\text{line}}} = \frac{V^0_{l}(F) - 0.1698\angle -170.72^\circ}{(Z^0_{l4})_{\text{line}}} \tag{4.30}$$

$$I^1_{l4(F)} = \frac{V^1_{l}(F) - V^1_{4}(F)}{(Z^1_{l4})_{\text{line}}} = \frac{V^1_{l}(F) - 0.5839\angle 1.3426^\circ}{(Z^1_{l4})_{\text{line}}} \tag{4.31}$$

$$I^2_{l4(F)} = \frac{V^2_{l}(F) - V^2_{4}(F)}{(Z^2_{l4})_{\text{line}}} = \frac{V^2_{l}(F) - 0.416\angle 178.11^\circ}{(Z^2_{l4})_{\text{line}}} \tag{4.32}$$

For line between bus 1 and bus 4 and applying Equations 4.30 through 4.32, with l = 1

$$\begin{aligned} I^0_{14(F)} &= \frac{V^0_{1}(F) - V^0_{4}(F)}{(Z^0_{14})_{\text{line}}} \\ &= \frac{0.08108\angle -163.355^\circ - 0.1698\angle -170.72^\circ}{(0.8 + j1.85)} \\ &= 0.0446\angle -63.96^\circ \end{aligned} \tag{4.33}$$

$$\begin{aligned} I^1_{14(F)} &= \frac{V^1_{1}(F) - V^1_{4}(F)}{(Z^1_{14})_{\text{line}}} \\ &= \frac{0.9278\angle -0.479^\circ - 0.5839\angle 1.3426^\circ}{(0.16 + j0.74)} \\ &= 0.455\angle -81.365^\circ \end{aligned} \tag{4.34}$$

$$\begin{aligned} I^2_{14(F)} &= \frac{V^2_{1}(F) - V^2_{4}(F)}{(Z^2_{14})_{\text{line}}} \\ &= \frac{0.1224\angle -176.368^\circ - 0.416\angle 178.11^\circ}{(0.16 + j0.74)} \\ &= 0.3888\angle -81.98^\circ \end{aligned} \tag{4.35}$$

From Equation 3.49

$$I_{14}^{abc}{}_{(F)} = AI_{14}^{012}{}_{(F)} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_{14\,(F)}^{0} \\ I_{14\,(F)}^{1} \\ I_{14\,(F)}^{2} \end{pmatrix} \tag{4.36}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \end{pmatrix} \begin{pmatrix} 0.0446\angle -63.96^\circ \\ 0.455\angle -81.365^\circ \\ 0.3888\angle -81.98^\circ \end{pmatrix}$$

$$= \begin{pmatrix} 0.0446\angle -63.96^\circ \\ 0.455\angle -81.365^\circ \\ 0.3888\angle -81.98^\circ \end{pmatrix}$$

The phase currents for line between Bus 1 and 4 are

$$\begin{aligned} I_{14\,(F)}^{a} &= 0.0446\angle -63.96^\circ \\ I_{14\,(F)}^{b} &= 0.455\angle -81.365^\circ \\ I_{14\,(F)}^{c} &= 0.388\angle -81.98^\circ \end{aligned} \tag{4.37}$$

For line between Bus 3 and Bus 4 and applying Equations 4.29 through 4.31 with l = 3

$$\begin{aligned} I_{34\,(F)}^{0} &= \frac{V_{3\,(F)}^{0} - V_{4\,(F)}^{0}}{(Z_{34}^{0})_{\text{line}}} \\ &= \frac{0\angle 0^\circ - 0.1698\angle -170.72^\circ}{(0)} = 0\angle 0^\circ \end{aligned} \tag{4.38}$$

$$\begin{aligned} I_{34\,(F)}^{1} &= \frac{V_{3\,(F)}^{1} - V_{4\,(F)}^{1}}{(Z_{34}^{1})_{\text{line}}} \\ &= \frac{0.6128\angle 2.513^\circ - 0.5839\angle 1.3426^\circ}{(j0.266)} = 0.1179\angle -65.15^\circ \end{aligned} \tag{4.39}$$

$$I_{34\,(F)}^{2} = \frac{V_{3\,(F)}^{2} - V_{4\,(F)}^{2}}{(Z_{34}^{2})_{\text{line}}} \tag{4.40}$$

$$= \frac{0.3889\angle 176.035^\circ - 0.416\angle 178.11^\circ}{(j0.266)} = 0.1156\angle -64.65^\circ$$

From Equation 3.49

$$\begin{aligned} I_{34}^{abc}(F) &= A I_{34}^{012}(F) \\ &= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_{34(F)}^{0} \\ I_{34(F)}^{1} \\ I_{34(F)}^{2} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \end{pmatrix} \begin{pmatrix} 0\angle 0^\circ \\ 0.1179\angle -65.15^\circ \\ 0.1156\angle -64.65^\circ \end{pmatrix} \end{aligned} \tag{4.41}$$

The sequence currents for line between Bus 3 and 4 are

$$\begin{aligned} I_{34(F)}^{a} &= 0.2335\angle -63.90^\circ \\ I_{34(F)}^{b} &= 0.1176\angle 116.07^\circ \\ I_{34(F)}^{c} &= 0.1159\angle 114.11^\circ \end{aligned} \tag{4.42}$$

For line between Bus 6 and Bus 4 and applying Equations 4.30 through 4.32 with l = 6

$$\begin{aligned} I_{64(F)}^{0} &= \frac{V_{6}^{0}(F) - V_{4}^{0}(F)}{(Z_{64}^{0})_{\text{line}}} \\ &= \frac{0.0408\angle -151.85^\circ - 0.1698\angle -170.72^\circ}{(0.9 + j2.06)} \\ &= 0.0586\angle -62.86^\circ \end{aligned} \tag{4.43}$$

$$\begin{aligned} I_{64(F)}^{1} &= \frac{V_{6}^{1}(F) - V_{4}^{1}(F)}{(Z_{64}^{1})_{\text{line}}} \\ &= \frac{0.7509\angle 0.0461^\circ - 0.5839\angle 1.3426^\circ}{(0.194 + j0.814)} \end{aligned} \tag{4.44}$$

$$= 0.2003\angle -81.068^\circ$$

$$I_{64(F)}^{2} = \frac{V_{6}^{2}{}_{(F)} - V_{4}^{2}{}_{(F)}}{(Z_{64}^{2})_{\text{line}}}$$

$$= \frac{0.249\angle 179.86^\circ - 0.416\angle 178.11^\circ}{(0.194 + j0.814)} \tag{4.45}$$

$$= 0.1999\angle -81.09^\circ$$

From Equation 3.49

$$I_{34}^{abc}{}_{(F)} = AI_{34}^{012}{}_{(F)}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_{34(F)}^{0} \\ I_{34(F)}^{1} \\ I_{34(F)}^{2} \end{pmatrix} \tag{4.46}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \end{pmatrix} \begin{pmatrix} 0.0586\angle -62.86^\circ \\ 0.2003\angle -81.068^\circ \\ 0.1999\angle -81.09^\circ \end{pmatrix}$$

The sequence currents for line between Bus 6 and 4 are

$$I_{64(F)}^{a} = 0.4562\angle -78.78^\circ$$
$$I_{64(F)}^{b} = 0.1455\angle 91.82^\circ \tag{4.47}$$
$$I_{64(F)}^{c} = 0.1457\angle 91.56^\circ$$

4.1.3 Line-to-Line Fault

The voltage at bus 4 before the fault is

$$V_{4(0)} = 1\angle 0^\circ \tag{4.48}$$

The sequence currents at bus 4 for a single line-to-ground fault are given by

$$I_{4}^{0} = 0 \tag{4.49}$$

$$I_{4}^{1} = \frac{V_{4(0)}}{(Z_{44}^{1} + Z_{44}^{2} + Z_{f})} \tag{4.50}$$

$$= \frac{1\angle 0^\circ}{2*(0.13269 + j0.57694) + 0} = 0.8446\angle -77.048^\circ$$

$$I_4^2 = -I_4^1 = 0.8446\angle 102.95^\circ \qquad (4.51)$$

The phase currents are

$$I_4^{abc}{}_{(F)} = AI_4^{012} \qquad (4.52)$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_4^0 \\ I_4^1 \\ I_4^2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \end{pmatrix} \begin{pmatrix} 0 \\ 0.8446\angle -77.048^\circ \\ 0.8446\angle 102.95^\circ \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1.4628\angle -167.05^\circ \\ 1.4628\angle 12.95^\circ \end{pmatrix}$$

The total fault current at Bus 4 is

$$\begin{aligned} I_4^b{}_{(F)} &= -I_4^c{}_{(F)} = -j\sqrt{3}I_4^1 \\ &= -j\sqrt{3}(0.8446\angle -77.048^\circ) \qquad (4.53) \\ &= 1.4628\angle -167.05^\circ \end{aligned}$$

$$I_4^c{}_{(F)} = -I_4^b{}_{(F)} = 1.4628\angle 12.952^\circ \qquad (4.54)$$

The sequence voltages are given by

$$\begin{aligned} V_i^0{}_{(F)} &= 0 - Z_{i4}^0 I_4^0 \\ V_i^1{}_{(F)} &= V_{i(0)} - Z_{i4}^1 I_4^1 \qquad (4.55) \\ V_i^2{}_{(F)} &= 0 - Z_{i4}^2 I_4^2 \end{aligned}$$

For Bus 1 then i = 1

$$V_1^0{}_{(F)} = 0 - Z_{14}^0 I_4^0$$
$$= 0 - (-0.0112 + j0.11474)(0\angle 0^\circ) \qquad (4.56)$$
$$= 0\angle 0^\circ$$

$$V_1^1{}_{(F)} = V_{1(0)} - Z_{14}^1 I_4^1$$
$$= 1.05\angle 0^\circ - (0.02254 + j0.17266)(0.8446\angle -77.048^\circ) \qquad (4.57)$$
$$= 0.9037\angle -0.895^\circ$$

$$V_1^2{}_{(F)} = 0 - Z_{14}^2 I_4^2$$
$$= 0 - (0.02254 + j0.17266)(0.8446\angle 102.95^\circ) \qquad (4.58)$$
$$= 0.147\angle 5.5123^\circ$$

For Bus 3 then i = 3

$$V_3^0{}_{(F)} = 0 - Z_{34}^0 I_4^0$$
$$= 0 - (0)(0\angle 0^\circ) \qquad (4.59)$$
$$= 0\angle 0^\circ$$

$$V_3^1{}_{(F)} = V_{3(0)} - Z_{34}^1 I_4^1$$
$$= 1\angle 0^\circ - (0.14333 + j0.53368)(0.8446\angle -77.048^\circ) \qquad (4.60)$$
$$= 0.533\angle 1.82^\circ$$

$$V_3^2{}_{(F)} = 0 - Z_{34}^2 I_4^2$$
$$= 0 - (0.14333 + j0.53368)(0.8446\angle 102.95^\circ) \qquad (4.61)$$
$$= 0.4667\angle -2.083^\circ$$

For Bus 4 then i = 4

$$V_4^0{}_{(F)} = 0 - Z_{44}^0 I_4^0$$
$$= 0 - (0.00756 + j0.24138)(0\angle 0^\circ) \qquad (4.62)$$
$$= 0\angle 0^\circ$$

$$V_4^1{}_{(F)} = V_{4(0)} - Z_{44}^1 I_4^1$$
$$= 1\angle 0^\circ - (0.13269 + j0.57694)(0.8446\angle -77.048^\circ) \qquad (4.63)$$
$$= 0.499\angle 0.002^\circ$$

$$V_{4\,(F)}^{2} = 0 - Z_{44}^{2} I_{4}^{2}$$
$$= 0 - (0.13269 + j0.57694)(0.8446\angle 102.95^{\circ}) \qquad (4.64)$$
$$= 0.5\angle -0.002^{\circ}$$

For Bus 6 then i = 6

$$V_{6\,(F)}^{0} = 0 - Z_{64}^{0} I_{4}^{0}$$
$$= 0 - (-0.01706 + j0.05554)(0\angle 0^{\circ}) \qquad (4.65)$$
$$= 0\angle 0^{\circ}$$

$$V_{6\,(F)}^{1} = V_{6\,(0)} - Z_{64}^{1} I_{4}^{1}$$
$$= 1\angle 0^{\circ} - (0.06881 + j0.34726)(0.8446\angle -77.048^{\circ}) \qquad (4.66)$$
$$= 0.7011\angle -0.742^{\circ}$$

$$V_{6\,(F)}^{2} = 0 - Z_{64}^{2} I_{4}^{2}$$
$$= 0 - (0.06881 + j0.34726)(0.8446\angle 102.95^{\circ}) \qquad (4.67)$$
$$= 0.299\angle 1.74^{\circ}$$

the line fault currents are given by

$$I_{l4\,(F)}^{0} = \frac{V_{l\,(F)}^{0} - V_{4\,(F)}^{0}}{(Z_{l4}^{0})_{\text{line}}} = \frac{V_{l\,(F)}^{0} - 0\angle 0^{\circ}}{(Z_{l4}^{0})_{\text{line}}} \qquad (4.68)$$

$$I_{l4\,(F)}^{1} = \frac{V_{l\,(F)}^{1} - V_{4\,(F)}^{1}}{(Z_{l4}^{1})_{\text{line}}} = \frac{V_{l\,(F)}^{1} - 0.499\angle 0.002^{\circ}}{(Z_{l4}^{1})_{\text{line}}} \qquad (4.69)$$

$$I_{l4\,(F)}^{2} = \frac{V_{l\,(F)}^{2} - V_{4\,(F)}^{2}}{(Z_{l4}^{2})_{\text{line}}} = \frac{V_{l\,(F)}^{2} - 0.5\angle -0.002^{\circ}}{(Z_{l4}^{2})_{\text{line}}} \qquad (4.70)$$

For the line between bus 1 and bus 4 and applying Equations 4.68 through 4.70, with l = 1

$$I_{14\,(F)}^{0} = \frac{V_{1\,(F)}^{0} - V_{4\,(F)}^{0}}{(Z_{14}^{0})_{\text{line}}} = \frac{0\angle 0^{\circ} - 0\angle 0^{\circ}}{(0.8 + j1.85)} = 0\angle 0^{\circ} \qquad (4.71)$$

$$I^{1}_{14\,(F)} = \frac{V^{1}_{1\,(F)} - V^{1}_{4\,(F)}}{(Z^{1}_{14})_{\text{line}}}$$

$$= \frac{0.9037\angle -0.895^{\circ} - 0.499\angle 0.002^{\circ}}{(0.16 + j0.74)} \quad (4.72)$$

$$= 0.535\angle -79.8^{\circ}$$

$$I^{2}_{14\,(F)} = \frac{V^{2}_{1\,(F)} - V^{2}_{4\,(F)}}{(Z^{2}_{14})_{\text{line}}}$$

$$= \frac{0.147\angle 5.5123^{\circ} - 0.5\angle -0.002^{\circ}}{(0.16 + j0.74)} \quad (4.73)$$

$$= 0.467\angle 99.91^{\circ}$$

From Equation 3.49

$$I^{abc}_{14\,(F)} = AI^{012}_{14\,(F)} \quad (4.74)$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I^{0}_{14\,(F)} \\ I^{1}_{14\,(F)} \\ I^{2}_{14\,(F)} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^{\circ} & 1\angle 120^{\circ} \\ 1 & 1\angle 120^{\circ} & 1\angle 240^{\circ} \end{pmatrix} \begin{pmatrix} 0\angle 0^{\circ} \\ 0.535\angle -79.8^{\circ} \\ 0.467\angle 99.91^{\circ} \end{pmatrix}$$

The phase currents for line between Bus 1 and 4 are

$$I^{a}_{14\,(F)} = 0.8856\angle -80.77^{\circ}$$
$$I^{b}_{14\,(F)} = 0.3777\angle -105.11^{\circ} \quad (4.75)$$
$$I^{c}_{14\,(F)} = 0.3895\angle -87.76^{\circ}$$

For line between bus 1 and bus 4 and applying Equations 4.68 through 4.70, with l = 3

$$I^{0}_{34\,(F)} = \frac{V^{0}_{3\,(F)} - V^{0}_{4\,(F)}}{(Z^{0}_{34})_{\text{line}}} = \frac{0\angle 0^{\circ} - 0\angle 0^{\circ}}{(0)} = 0\angle 0^{\circ} \quad (4.75)$$

$$I_{34}^{1}(F) = \frac{V_3^1(F) - V_4^1(F)}{(Z_{34}^{1})_{\text{line}}}$$

$$= \frac{0.533\angle 1.82^\circ - 0.499\angle 0.002^\circ}{(j0.266)} \tag{4.76}$$

$$= 0.1418\angle -63.373^\circ$$

$$I_{34}^{2}(F) = \frac{V_3^2(F) - V_4^2(F)}{(Z_{34}^{2})_{\text{line}}}$$

$$= \frac{0.4667\angle -2.083^\circ - 0.5\angle -0.002^\circ}{(j0.266)} \tag{4.77}$$

$$= 0.141\angle 116.7570^\circ$$

From Equation 3.49

$$I_{34}^{abc}(F) = AI_{34}^{012}(F) \tag{4.78}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_{34}^{0}(F) \\ I_{34}^{1}(F) \\ I_{34}^{2}(F) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \end{pmatrix} \begin{pmatrix} 0\angle 0^\circ \\ 0.1418\angle -63.37^\circ \\ 0.141\angle 116.76^\circ \end{pmatrix}$$

The phase currents for line between Bus 3 and 4 are

$$\begin{aligned} I_{34}^{a}(F) &= 0.0009\angle -85.61^\circ \\ I_{34}^{b}(F) &= 0.2447\angle -153.4^\circ \\ I_{34}^{c}(F) &= 0.2450\angle -26.78^\circ \end{aligned} \tag{4.79}$$

For line between bus 6 and bus 4 and applying Equations 4.68 through 4.70, with l = 6

$$I_{64}^{0}(F) = \frac{V_6^0(F) - V_4^0(F)}{(Z_{64}^{0})_{\text{line}}} = \frac{0\angle 0^\circ - 0\angle 0^\circ}{(0.9 + j2.06)} = 0\angle 0^\circ \tag{4.80}$$

$$I_{64}^{1}(F) = \frac{V_6^1(F) - V_4^1(F)}{(Z_{64}^1)_{\text{line}}}$$

$$= \frac{0.7011\angle -0.742^\circ - 0.499\angle 0.002^\circ}{(0.194 + j0.814)} \tag{4.81}$$

$$= 0.242\angle -79.17^\circ$$

$$I_{64}^{2}(F) = \frac{V_6^2(F) - V_4^2(F)}{(Z_{64}^2)_{\text{line}}}$$

$$= \frac{0.299\angle 1.74^\circ - 0.5\angle -0.002^\circ}{(0.194 + j0.814)} \tag{4.82}$$

$$= 0.240\angle 100.810^\circ$$

From Equation 3.49

$$I_{64}^{abc}(F) = AI_{64}^{012}(F) \tag{4.83}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_{34}^0(F) \\ I_{34}^1(F) \\ I_{34}^2(F) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \end{pmatrix} \begin{pmatrix} 0\angle 0^\circ \\ 0.242\angle -79.17^\circ \\ 0.240\angle 100.81^\circ \end{pmatrix}$$

The phase currents for line between Bus 6 and 4 are

$$\begin{aligned} I_{64}^{a}(F) &= 0\angle 0^\circ \\ I_{64}^{b}(F) &= 0.242\angle -79.17^\circ \\ I_{64}^{c}(F) &= 0.240\angle 100.81^\circ \end{aligned} \tag{4.84}$$

4.1.4 Double Line-to-Ground Fault

The sequence currents at bus 4 for a single line-to-ground fault are given by

$$I_4^1 = \frac{V_{4(0)}}{(Z_{44}^1 \; + \; Z_{44}^2 \dfrac{Z_{44}^0 + 3Z_f}{Z_{44}^2 + Z_{44}^0 + 3Z_f})} \tag{4.85}$$

$$I_4^1 = \frac{1\angle 0^\circ}{[(0.592\angle 77.048^\circ) \; + \; \dfrac{(0.592\angle 77.048^\circ)(0.241\angle 88.206^\circ)}{(0.592\angle 77.048^\circ) + (0.241\angle 88.206^\circ)}]} \tag{4.86}$$
$$= 1.3107\angle -78.83^\circ$$

$$I_4^2 = \frac{V_{4(0)} - Z_{44}^1 I_4^1}{Z_{44}^2}$$
$$= \frac{1\angle 0^\circ - (0.13269 + j0.57694)(1.3107\angle -78.83^\circ)}{(0.13269 + j0.57694)} \tag{4.87}$$
$$= \frac{0.225\angle 6.136^\circ}{0.592\angle 77.048^\circ}$$
$$= 0.3813\angle -70.91^\circ \,/\, 109.105^\circ$$

$$I_4^0 = \frac{V_{4(0)} - Z_{44}^1 I_4^1}{(Z_{44}^0 + 3Z_f)}$$
$$= \frac{1\angle 0^\circ - (0.13269 + j0.57694)(1.3107\angle -78.83^\circ)}{(0.00756 + j0.24138)} \tag{4.88}$$
$$= \frac{0.225\angle 6.136^\circ}{0.241\angle 88.206^\circ}$$
$$= 0.9347\angle -82.07^\circ \,/\, 97.94^\circ$$

The phase currents are

$$I_{4\,(F)}^{abc} = AI_4^{012} \tag{4.89}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_4^0 \\ I_4^1 \\ I_4^2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \end{pmatrix} \begin{pmatrix} 0.9347\angle 97.94^\circ \\ 1.3107\angle -78.83^\circ \\ 0.3813\angle 109.105^\circ \end{pmatrix}$$

$$= \begin{pmatrix} 0.0002\angle 91.52^\circ \\ 1.935\angle 146.77^\circ \\ 2.113\angle 54.33^\circ \end{pmatrix}$$

The total fault current at Bus 4 is

$$\begin{aligned} I_{4(F)} &= I_{4(F)}^{b} + I_{4(F)}^{c} \\ &= 1.935\angle 146.77^\circ + 2.113\angle 54.33^\circ = 2.803\angle 97.92^\circ \end{aligned} \tag{4.90}$$

The sequence voltages are given by

$$\begin{aligned} V_{i(F)}^{0} &= 0 - Z_{i4}^{0} I_{4}^{0} \\ V_{i(F)}^{1} &= V_{i(0)} - Z_{i4}^{1} I_{4}^{1} \\ V_{i(F)}^{2} &= 0 - Z_{i4}^{2} I_{4}^{2} \end{aligned} \tag{4.91}$$

For Bus 1 then i = 1

$$\begin{aligned} V_{1(F)}^{0} &= 0 - Z_{14}^{0} I_{4}^{0} \\ &= 0 - (-0.0112 + j0.11474)(0.9347\angle 97.94^\circ) \\ &= 0.108\angle 13.51^\circ \end{aligned} \tag{4.92}$$

$$\begin{aligned} V_{1(F)}^{1} &= V_{1(0)} - Z_{14}^{1} I_{4}^{1} \\ &= 1.05\angle 0^\circ - (0.02254 + j0.17266)(1.3107\angle -78.83^\circ) \\ &= 0.822\angle -1.035^\circ \end{aligned} \tag{4.93}$$

$$\begin{aligned} V_{1(F)}^{2} &= 0 - Z_{14}^{2} I_{4}^{2} \\ &= 0 - (0.02254 + j0.17266)(0.3813\angle 109.105^\circ) \\ &= 0.066\angle 11.66^\circ \end{aligned} \tag{4.94}$$

For Bus 3 then i = 3

$$V_3^0{}_{(F)} = 0 - Z_{34}^0 I_4^0$$
$$= 0 - (0)(0.9347\angle 97.94^\circ) \quad (4.95)$$
$$= 0\angle 0^\circ$$
$$V_3^1{}_{(F)} = V_{3(0)} - Z_{34}^1 I_4^1$$
$$= 1\angle 0^\circ - (0.14333 + j0.53368)(1.3107\angle -78.83^\circ) \quad (4.96)$$
$$= 0.282\angle 9.98^\circ$$

$$V_3^2{}_{(F)} = 0 - Z_{34}^2 I_4^2$$
$$= 0 - (0.14333 + j0.53368)(0.3813\angle 109.105^\circ) \quad (4.97)$$
$$= 0.21\angle 4.07^\circ$$

For Bus 4 then i = 4

$$V_4^0{}_{(F)} = 0 - Z_{44}^0 I_4^0$$
$$= 0 - (0.00756 + j0.24138)(0.9347\angle 97.94^\circ) \quad (4.98)$$
$$= 0.226\angle 6.146^\circ$$

$$V_4^1{}_{(F)} = V_{4(0)} - Z_{44}^1 I_4^1$$
$$= 1\angle 0^\circ - (0.13269 + j0.57694)(1.3107\angle -78.83^\circ) \quad (4.99)$$
$$= 0.226\angle 6.137^\circ$$

$$V_4^2{}_{(F)} = 0 - Z_{44}^2 I_4^2$$
$$= 0 - (0.13269 + j0.57694)(0.3813\angle 109.105^\circ) \quad (4.100)$$
$$= 0.226\angle 6.152^\circ$$

For Bus 6 then i = 6

$$V_6^0{}_{(F)} = 0 - Z_{64}^0 I_4^0$$
$$= 0 - (-0.01706 + j0.05554)(0.9347\angle 97.94^\circ) \quad (4.101)$$
$$= 0.054\angle 25.02^\circ$$

$$V_6^1{}_{(F)} = V_{6(0)} - Z_{64}^1 I_4^1$$
$$= 1\angle 0^\circ - (0.06881 + j0.34726)(1.3107\angle -78.83^\circ) \quad (4.102)$$
$$= 0.536\angle 0.033^\circ$$

$$\begin{aligned} V_6^2(F) &= 0 - Z_{64}^2 I_4^2 \\ &= 0 - (0.06881 + j0.34726)(0.3813\angle 109.105^\circ) \\ &= 0.135\angle 7.897^\circ \end{aligned} \tag{4.103}$$

the line fault currents are given by

$$I_{l4}^0(F) = \frac{V_l^0(F) - V_4^0(F)}{(Z_{l4}^0)_{\text{line}}} = \frac{V_l^0(F) - 0.226\angle 6.146^\circ}{(Z_{l4}^0)_{\text{line}}} \tag{4.104}$$

$$I_{l4}^1(F) = \frac{V_l^1(F) - V_4^1(F)}{(Z_{l4}^1)_{\text{line}}} = \frac{V_l^1(F) - 0.226\angle 6.137^\circ}{(Z_{l4}^1)_{\text{line}}} \tag{4.105}$$

$$I_{l4}^2(F) = \frac{V_l^2(F) - V_4^2(F)}{(Z_{l4}^2)_{\text{line}}} = \frac{V_l^2(F) - 0.226\angle 6.152^\circ}{(Z_{l4}^2)_{\text{line}}} \tag{4.106}$$

For line between bus 1 and bus 4 and applying Equations 4.104 through 4.106, with l = 1

$$\begin{aligned} I_{14}^0(F) &= \frac{V_1^0(F) - V_4^0(F)}{(Z_{14}^0)_{\text{line}}} \\ &= \frac{0.108\angle 13.51^\circ - 0.226\angle 6.146^\circ}{(0.8 + j1.85)} = 0.059\angle 112.89^\circ \end{aligned} \tag{4.107}$$

$$\begin{aligned} I_{14}^1(F) &= \frac{V_1^1(F) - V_4^1(F)}{(Z_{14}^1)_{\text{line}}} \\ &= \frac{0.822\angle -1.035^\circ - 0.226\angle 6.137^\circ}{(0.16 + j0.74)} = 0.79\angle -81.53^\circ \end{aligned} \tag{4.108}$$

$$\begin{aligned} I_{14}^2(F) &= \frac{V_1^2(F) - V_4^2(F)}{(Z_{14}^2)_{\text{line}}} \\ &= \frac{0.066\angle 11.66^\circ - 0.226\angle 6.152^\circ}{(0.16 + j0.74)} = 0.212\angle 111.27^\circ \end{aligned} \tag{4.109}$$

From Equation 3.49

$$I_{14}^{abc}{}_{(F)} = AI_{14}^{012}{}_{(F)} \tag{4.110}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_{14(F)}^{0} \\ I_{14(F)}^{1} \\ I_{14(F)}^{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \end{pmatrix} \begin{pmatrix} 0.059\angle 112.89^\circ \\ 0.79\angle -81.53^\circ \\ 0.212\angle 111.27^\circ \end{pmatrix}$$

$$= \begin{pmatrix} 0.5297\angle -88.21^\circ \\ 0.9082\angle 168.64^\circ \\ 0.9550\angle 32.53^\circ \end{pmatrix}$$

The phase currents for line between Bus 1 and 4 are

$$\begin{aligned} I_{14(F)}^{a} &= 0.5297\angle -88.21^\circ \\ I_{14(F)}^{b} &= 0.9082\angle -168.64^\circ \\ I_{14(F)}^{c} &= 0.9550\angle -32.53^\circ \end{aligned} \tag{4.111}$$

For line between bus 3 and bus 4 and applying Equations 4.104 through 4.106, with l = 3

$$\begin{aligned} I_{34(F)}^{0} &= \frac{V_3^0{}_{(F)} - V_4^0{}_{(F)}}{(Z_{34}^0)_{\text{line}}} \\ &= \frac{0\angle 0^\circ - 0.226\angle 6.146^\circ}{(0)} = 0\angle 0^\circ \end{aligned} \tag{4.112}$$

$$\begin{aligned} I_{34(F)}^{1} &= \frac{V_3^1{}_{(F)} - V_4^1{}_{(F)}}{(Z_{34}^1)_{\text{line}}} \\ &= \frac{0.282\angle 9.98^\circ - 0.226\angle 6.137^\circ}{(j0.266)} = 0.22\angle -65.01^\circ \end{aligned} \tag{4.113}$$

$$I_{34}^{2}(F) = \frac{V_3^2(F) - V_4^2(F)}{(Z_{34}^2)_{\text{line}}}$$
$$= \frac{0.21\angle 4.07^\circ - 0.226\angle 6.152^\circ}{(j0.266)} = 0.067\angle 121.45^\circ \qquad (4.114)$$

From Equation 3.49

$$I_{34}^{abc}(F) = AI_{34}^{012}(F) \qquad (4.115)$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_{34}^0(F) \\ I_{34}^1(F) \\ I_{34}^2(F) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \end{pmatrix} \begin{pmatrix} 0\angle 0^\circ \\ 0.22\angle -65.01^\circ \\ 0.067\angle 121.45^\circ \end{pmatrix}$$

$$= \begin{pmatrix} 0.1536\angle -67.83^\circ \\ 0.2543\angle -171.03^\circ \\ 0.2653\angle 43.27^\circ \end{pmatrix}$$

The phase currents for line between Bus 1 and 4 are

$$\begin{aligned} I_{34}^{a}(F) &= 0.1536\angle -67.83^\circ \\ I_{34}^{b}(F) &= 0.2543\angle -171.03^\circ \\ I_{34}^{c}(F) &= 0.2653\angle 43.27^\circ \end{aligned} \qquad (4.116)$$

For line between bus 6 and bus 4 and applying Equations 4.104 through 4.106, with l = 6

$$I_{64}^{0}(F) = \frac{V_6^0(F) - V_4^0(F)}{(Z_{64}^0)_{\text{line}}}$$
$$= \frac{0.054\angle 25.02^\circ - 0.226\angle 6.146^\circ}{(0.9 + j2.06)} = 0.0782\angle 114.04^\circ \qquad (4.117)$$

$$I_{64(F)}^{1} = \frac{V_{6\,(F)}^{1} - V_{4\,(F)}^{1}}{(Z_{64}^{1})_{\text{line}}}$$

$$= \frac{0.536\angle 0.033^{\circ\circ} - 0.226\angle 6.137^{\circ}}{(0.194 + j0.814)} = 0.373\angle -80.97^{\circ} \quad (4.117)$$

$$I_{64(F)}^{2} = \frac{V_{6\,(F)}^{2} - V_{4\,(F)}^{2}}{(Z_{64}^{2})_{\text{line}}}$$

$$= \frac{0.135\angle 7.897^{\circ} - 0.226\angle 6.152^{\circ}}{(0.194 + j0.814)} = 0.109\angle 106.97^{\circ} \quad (4.118)$$

From Equation 3.49

$$I_{64}^{abc}{}_{(F)} = AI_{64}^{012}{}_{(F)} \quad (4.119)$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_{64(F)}^{0} \\ I_{64(F)}^{1} \\ I_{64(F)}^{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^{\circ} & 1\angle 120^{\circ} \\ 1 & 1\angle 120^{\circ} & 1\angle 240^{\circ} \end{pmatrix} \begin{pmatrix} 0.0782\angle 114.04^{\circ} \\ 0.373\angle -80.97^{\circ} \\ 0.109\angle 106.97^{\circ} \end{pmatrix}$$

$$= \begin{pmatrix} 0.1928\angle -91.52^{\circ} \\ 0.4714\angle 164.59^{\circ} \\ 0.4603\angle 37.73^{\circ} \end{pmatrix}$$

The phase currents for line between Bus 6 and 4 are

$$\begin{aligned} I_{64(F)}^{a} &= 0.1928\angle -91.52^{\circ} \\ I_{64(F)}^{b} &= 0.4714\angle -164.59^{\circ} \\ I_{64(F)}^{c} &= 0.4603\angle 37.73^{\circ} \end{aligned} \quad (4.120)$$

4.2 Matlab and Simulation

Since symmetrical components method includes many matrix operations, computer can be utilized to perform fault analysis in well-organized, effective, faster and logical means. The goal of including this part is to confirm the hand calculations found previously. In addition, the code can be used to accomplish this task where hand calculations can't handle a larger system and the analysis become difficult.

Matlab was selected as the simulation tool in this project due to several reasons. Our background of Matlab was the main reason behind this choice. In addition, any code can be edited and modified easily to handle any future cases using the command edit window. Also, Matlab contains many built-in functions to resolve different electrical problem. Matlab has also many unique features that allow users to develop an algorithm in order to resolve any specific case. These features will not be discussed in this part. However, a short introduction will be provided about Matlab since it the most common used electrical program in general.

Matlab is high-ranking software package where its mathematical calculations are done using matrix operations. It was driven from the description "Matrix Laboratory" [S11] the program was created in 1984 by Mathworks who established afterward a community called Matlab Central [S11]. Most of that community' members are engineers from all around the world. Users can write their codes using the M-files available in Matlab. M-files is a script that will perform series of commands. This script can be edited and modified based on the user's needs at any stage. The script will perform specific mathematical operations using given data and provide output solutions.

The Matlab code used to solve the problem statement in this project starts by identifying the system input arguments. These variables are manly reactance of the system transformers, branches and generators. This step is to form the system admittance. After that, the fault analysis calculations were done for the three-phase, single line-to-ground, line-to-line and double line-to-ground faults using the pre-defined functions from Hadi Saadat's known codes. The code can be seen in Appendix B with the output results which will be discussed in the next chapter.

4.3 Results and Discussion

In this section, the results of hand calculations and the Matlab code using symmetrical components method will be discussed. This discussion will be done for all types of fault listed earlier. Then, the comments and recommendations will be provided based on that. The accuracy of the hand calculations will be confirmed by comparing its results with the code's output. The system consists of six buses, two generators and seven branches as seen in Figure 4.1. All data was provided in Appendix A. The results are given in the following tables.

4.3.1 Three-Phase Fault

Table 4.1 Bus voltages for three-phase fault

Bus #	Hand Calculation		MATLAB		Paul Anderson's Solution	
	Magnitude p.u.	Angle °	Magnitude p.u.	Angle °	Magnitude p.u.	Angle °
4	0.0000	0.000	0.0000	0.0000	0.0000	0.000
1	0.7578	-2.150	0.7432	-2.2887	0.7078	-2.300
3	0.0752	26.600	0.0753	26.7700	0.0752	26.800
6	0.4027	-2.600	0.4027	-2.5905	0.4027	-2.600

Table 4.2 Fault currents for three-phase fault

	Hand Calculation		MATLAB		Paul Anderson's Solution	
	Magnitude p.u.	Angle °	Magnitude p.u.	Angle °	Magnitude p.u.	Angle °
Bus 4	1.6890	-77.040	1.6892	-77.05	1.6890	-77.000
1 to 4	1.0000	-79.900	0.9816	-80.09	0.9350	-80.100
3 to 4	0.2827	-63.400	0.2829	-63.23	0.2830	-63.200
6 to 4	0.4810	-79.190	0.4812	-79.18	0.4810	-79.200

4.3.2 Single Line-to-Ground Fault

Table 4.3 Bus sequence voltages for single line-to-ground fault

Bus #		Hand Calculation		MATLAB	
		Magnitude p.u.	Angle °	Magnitude p.u.	Angle °
Positive Sequence	4	0.5842	1.340	0.5840	1.340
	1	0.9278	-0.479	0.9278	-0.479
	3	0.6130	2.5100	0.6128	2.510
	6	0.7510	0.0400	0.7510	0.046
Negative Sequence	4	0.4162	178.120	0.4164	178.110
	1	0.1224	-176.37	0.1225	-176.580
	3	0.3880	176.00	0.3887	176.030
	6	0.2490	179.90	0.2490	179.860
Zero Sequence	4	0.1698	-170.70	0.1699	-170.720
	1	0.0810	-163.35	0.0810	-163.360
	3	0.0000	0.0000	0.0000	0.000
	6	0.0408	-151.84	0.0409	-151.850

Table 4.4 Fault sequence currents for single line-to-ground fault

		Hand Calculation		MATLAB	
		Magnitude p.u.	Angle °	Magnitude p.u.	Angle °
Positive Sequence	Bus 4	0.703	-78.93	0.703	-78.93
	1 to 4	0.460	-83.60	0.455	-81.37
	3 to 4	0.118	-65.10	0.118	-65.11
	6 to 4	0.200	-81.10	0.200	-81.07
Negative Sequence	Bus 4	0.703	-78.93	0.703	-78.93
	1 to 4	0.389	-82.00	0.389	-81.97
	3 to 4	0.119	-65.10	0.118	-65.11
	6 to 4	0.200	-81.10	0.200	-81.07
Zero Sequence	Bus 4	0.703	-78.93	0.703	-78.93
	1 to 4	0.045	-63.90	0.045	-63.95
	3 to 4	0.000	0.00	0.000	0.00
	6 to 4	0.058	-62.84	0.058	-62.91
Fault Currents	Bus 4	2.109	-78.93	2.110	-78.93
	1 to 4	0.891	-81.90	0.887	-80.76
	3 to 4	0.237	-65.10	0.236	-65.11
	6 to 4	0.452	-77.60	0.457	-78.79

4.3.3 Double Line-to-Ground Fault

Table 4.5 Bus sequence voltages for double line-to-ground fault

Bus #		Hand Calculation		MATLAB	
		Magnitude p.u.	Angle °	Magnitude p.u.	Angle °
Positive Sequence	Bus 4	0.226	6.137	0.226	6.147
	Bus 1	0.822	-1.035	0.822	-1.034
	Bus 3	0.282	9.98	0.282	9.986
	Bus 6	0.536	0.03	0.536	0.035
Negative Sequence	Bus 4	0.226	6.152	0.226	6.147
	Bus 1	0.066	11.66	0.064	11.661
	Bus 3	0.210	4.07	0.211	4.066
	Bus 6	0.135	7.90	0.135	7.891
Zero Sequence	Bus 4	0.226	6.146	0.226	6.147
	Bus 1	0.108	13.51	0.108	13.51
	Bus 3	0.000	0.00	0.000	0.00
	Bus 6	0.054	25.02	0.0543	25.02

Table 4.6 Fault sequence currents for double line-to-ground fault

		Hand Calculation		MATLAB	
		Magnitude p.u.	Angle °	Magnitude p.u.	Angle °
Positive Sequence	Bus 4	1.311	-78.83	1.3107	-78.83
	1 to 4	0.790	-81.53	0.791	-81.53
	3 to 4	0.220	-65.01	0.219	-65.0
	6 to 4	0.373	-80.97	0.373	-80.97
Negative Sequence	Bus 4	0.381	-70.91	0.3812	-72.03
	1 to 4	0.212	111.27	0.211	106.06
	3 to 4	0.067	121.45	0.064	122.9
	6 to 4	0.109	106.97	0.109	106.97
Zero Sequence	Bus 4	0.935	-82.07	0.934	-81.59
	1 to 4	0.059	112.89	0.059	112.89
	3 to 4	0.000	0.00	0.000	0.00
	6 to 4	0.078	114.04	0.078	113.99
Fault Currents	Bus 4	2.803	97.92	2.803	97.94
	1 to 4	0.5297	-88.21	0.526	-86.167
	3 to 4	0.1536	-67.82	0.1565	-68.23
	6 to 4	0.1929	-91.49	0.1929	-91.48

4.3.4 Line-to-Line Fault

Table 4.7 Bus sequence voltages for line-to-line fault

Bus #		Hand Calculation		MATLAB	
		Magnitude p.u.	Angle °	Magnitude p.u.	Angle °
Positive Sequence	Bus 4	0.499	0.002	0.500	0.00
	Bus 1	0.904	-0.90	0.904	-0.90
	Bus 3	0.533	1.82	0.534	1.82
	Bus 6	0.701	-0.74	0.701	-0.74
Negative Sequence	Bus 4	0.500	-0.002	0.500	0.00
	Bus 1	0.147	5.51	0.147	5.51
	Bus 3	0.467	-2.08	0.467	-2.08
	Bus 6	0.299	1.74	0.299	1.74
Zero Sequence	Bus 4	0.000	0.00	0.000	0.00
	Bus 1	0.000	0.00	0.000	0.00
	Bus 3	0.000	0.00	0.000	0.00
	Bus 6	0.000	0.00	0.000	0.00

Table 4.8 Fault sequence currents for line-to-line fault

		Hand Calculation		MATLAB	
		Magnitude p.u.	Angle °	Magnitude p.u.	Angle °
Positive Sequence	Bus 4	0.845	-77.05	0.844	-77.04
	1 to 4	0.535	-79.80	0.533	-79.81
	3 to 4	0.142	-63.37	0.141	-63.23
	6 to 4	0.242	-79.17	0.241	-79.19
Negative Sequence	Bus 4	0.845	102.95	0.844	102.95
	1 to 4	0.467	99.91	0.467	99.91
	3 to 4	0.141	116.76	0.141	116.76
	6 to 4	0.240	100.81	0.241	100.80
Zero Sequence	Bus 4	0.000	0.00	0.000	0.00
	1 to 4	0.000	0.00	0.000	0.00
	3 to 4	0.000	0.00	0.000	0.00
	6 to 4	0.000	0.00	0.000	0.00
Fault Currents	Bus 4	0.000	0.00	0.0001	12.95
	1 to 4	0.068	-77.81	0.066	-77.83
	3 to 4	0.0001	-153.27	0.0002	-152.3
	6 to 4	0.002	-76.77	0.0001	-76.79

From the above tables, it can be seen that the results from the simulation output matches the hand calculations results provided earlier with small error. The error does not exceed 0.1 per unit value. In addition, both results confirmed the final results of the system analysis provided in Paul Anderson’s Book. Also, the admittance matrix formed in the hand calculations’ part is almost identical to the matrix generated by Matlab using Hadi Saadat’s codes.

As seen from the above tables which also confirm the content of [11], the outcome of the simulation stage in this project can be listed in the following points:

1 In three-phase fault, the voltages at faulted bus phases dropped to zero during the fault. The faulted bus is bus number four where Phase A, B and C has a zero voltage potential.

2 However, only voltage at Phase A is equal to zero in single line-to-ground fault. In addition, only Phase A has current since it is the faulted phase in this type of fault as we assumed earlier in the mathematical model. This current is the second highest fault currents of all types.

3 Since Phase B and Phase C are in contact in line-to-line fault, the voltages at both phases are equal. The fault current are passing from B to C. in Phase A, the current is equal to zero compared to the fault current.

4 In double line-to-ground fault, Phase B and C voltages are equal to zero. The faulted current is flowing through both phases only. In addition, this type of fault is the most sever fault on the system which can be seen from its current value.

Chapter 5

CONCLUSIONS

In this project, fault analysis was done for a 6-bus system where bus number four was the main focus of this report. The analysis was carried manually using symmetrical components method. The results were compared with software solutions to validate the hand calculations' accuracy. The error was acceptable and below 0.1 p.u. for all type of fault as discussed in the previous chapter. The following observations have been made based on the results obtained from the analysis:

1 In three-phase fault, the voltages at faulted bus phases dropped to zero during the fault. The faulted bus is bus number four where Phase A, B and C has a zero voltage potential.

2 However, only voltage at Phase A is equal to zero in single line-to-ground fault. In addition, only Phase A has current since it is the faulted phase in this type of fault as we assumed earlier in the mathematical model. This current is the second highest fault currents of all types.

3 Since Phase B and Phase C are in contact in line-to-line fault, the voltages at both phases are equal. The fault current are passing from B to C. in Phase A, the current is equal to zero compared to the fault current.

4 In double line-to-ground fault, Phase B and C voltages are equal to zero. The faulted current is flowing through both phases only. In addition, this type of

fault is the most sever fault on the system which can be seen from its current value.

APPENDIX A

Data for 6-Node Network

6-Node network with impedances in pu on a 50 MVA Base

Impedance Number	Connecting Nodes	Self Impedance		Mutual Impedance	
		Z1=Z2	Z0	ZM	Branch
1	1-6	0.123+j0.518	0.492+j1.042		
2	1-4	0.080+j0.370	0.400+j0.925	0.250+j0.475	3
3	4-6	0.097+j0.407	0.450+j1.030	0.250+j0.476	2
4	5-6	0.000+j0.300	0.000+j0.300		
5	2-5	0.282+j0.640	1.410+j1.920		
6	2-3	0.723+j1.050	1.890+j2.630		
7	3-4	0.000+j0.133	0.000+j0.133		
8	0-4	0.000-j34.100			
9	0-1	0.000-j29.500			
10	0-6	0.000-j28.500			
11	Gen 1	0.010+j0.120			
12	Gen 2	0.015+j0.240	0.000+j0.016		

Transmission and Transformer data assembly

Line			Actual			Converted no tap effect		
P	Q	NO.	R(%)	X(%)	KVAC	BC/2 (pu)	G(pu)	B(pu)
1	4	0	16.00	74.00	1406.80	0.0070	0.2793	-1.2910
1	6	0	24.60	103.60	1983.00	0.0099	0.2170	-0.9137
2	3	0	144.60	210.00	0.00	0.0000	0.2224	-0.3230
2	5	0	56.40	128.00	0.00	0.0000	0.2383	-0.6542
4	3	0	0.00	26.60	0.00	0.0000	0.0000	-3.7594
4	6	0	19.40	81.40	1525.80	0.0076	0.2771	-1.1625
6	5	0	0.00	60.00	0.00	0.0000	0.0000	-1.6667

Transmission and Transformer data assembly

Line							
			MVA Rating	TAP Ratio	Tap Limits		Phase shift
P	Q	NO.			TMIN	TMAX	
1	4	0	0				0.0
1	6	0	0				0.0
2	3	0	0				0.0
2	5	0	0				0.0
4	3	0	0	0.909			0.0
4	6	0	0				0.0
6	5	0	0	0.976			0.0

Transmission and Transformer data assembly

Line			Map Data						
			Flow			TAP	REV	FLOW	
P	Q	NO.	PG	LOC	Q	LOC	PG	LOC	Q
1	4	0	0	0			0	0	
1	6	0	0	0			0	0	
2	3	0	0	0			0	0	
2	5	0	0	0			0	0	
4	3	0	0	0		0	0	0	
4	6	0	0	0			0	0	
6	5	0	0	0		0	0	0	

Base case bus data entered

Bus			Voltage			Load	
No.	Name	Area	Reg	Mag (pu)	Ang(deg)	MW	MVAR
1	One	64	2	1.050	0.0	0.0	0.0
2	Two	64	1	1.100	0.0	0.0	0.0
3	Three	64	0	1.000	0.0	27.5	6.5
4	Four	64	0	1.000	0.0	0.0	0.0
5	Five	64	0	1.000	0.0	15.0	9.0
6	Six	64	0	1.000	0.0	25.0	2.5

Base case bus data entered

Bus			Generation		QMIN MVAR	QMAX MVAR	Reactor KVAR
No.	Name	Area	MW	MVAR			
1	One	64	0.0	0.0	0.0	0.0	0
2	Two	64	25.0	0.0	-12.5	12.5	0
3	Three	64	0.0	0.0	0.0	0.0	0
4	Four	64	0.0	0.0	0.0	0.0	0
5	Five	64	0.0	0.0	0.0	0.0	0
6	Six	64	0.0	0.0	0.0	0.0	0

Base case bus data entered

Bus			Map Data				
				Volt	Load	Gen	Reactor
No.	Name	Area	PAGE	LOC A	LOC Q	LOC Q	LOC QS
1	One	64	0	0	0	0	0
2	Two	64	0	0	0	0	0
3	Three	64	0	0	0	0	0
4	Four	64	0	0	0	0	0
5	Five	64	0	0	0	0	0
6	Six	64	0	0	0	0	0

Summary

Line and Bus Totals		Actual	Max		MW	MVAR
Transmission Lines		5	750	Total Load	67.500	18.000
Transformers	Fixed	2	250	Total Losses	5.112	17.666
	LTC	0	250	Line Charging		-4.618
Phase Shifters		0	0	Fixed Cao/Reac		0.0
Total Lines		7	750	System Mismatch	-0.002	-0.001
Buses	Non reg (Including Swing)	5	500	Total Generation	72.610	31.047
	Generator	1	190			
Total Buses		6	500			

Data for positive sequence (100 MVA base)

Self								
Actual					Converted			
P	Q	NO.	R(%)	X(%)	R(pu)	X(pu)	G(pu)	B(pu)
0	1	0	2.00	24.00	0.0200	0.2400	0.3448	-4.1379
0	2	0	3.00	48.00	0.0300	0.4800	0.1297	-2.0752
1	4	0	16.00	74.00	0.1600	0.7400	0.2791	-1.2910
1	6	0	24.60	103.60	0.2460	1.0360	0.2170	-0.9137
2	3	0	144.60	210.00	1.4460	2.1000	0.2224	-0.3230
2	5	0	56.40	128.00	0.5640	1.2800	0.2883	-0.6542
3	4	0	0.00	26.60	0.0000	0.2660	0.0000	-3.7594
6	4	0	19.40	81.40	0.1940	0.8140	0.2771	-1.1625
5	6	0	0.00	60.00	0.0000	0.6000	0.0000	-1.6667

Existing Z matrix for the positive and negative sequence network

Bus	Bus	R	X
1	1	0.02253	0.21503
1	2	-0.00609	0.04974
1	3	0.02635	0.16117
1	4	0.02254	0.17266
1	5	0.02118	0.12825
1	6	0.01831	0.16379
2	2	0.04422	0.38094
2	3	-0.01594	0.15713
2	4	-0.00786	0.13434
2	5	-0.00698	0.22306
2	6	0.00023	0.15223
3	3	0.16244	0.73912
3	4	0.14333	0.53368
3	5	0.06192	0.27007
3	6	0.07295	0.32786
4	4	0.13269	0.57694
4	5	0.06506	0.27818
4	6	0.06881	0.34726
5	5	0.16569	0.80649
5	6	0.13256	0.46538
6	6	0.13034	0.61119

Data for Zero sequence (100 MVA base)

Self								
Actual					Converted			
P	Q	NO.	R(%)	X(%)	R(pu)	X(pu)	G(pu)	B(pu)
0	2	0	0.00	3.20	0.0000	0.0320	0.0000	-31.2500
0	4	0	0.00	26.60	0.0000	0.2660	0.0000	-3.7594
0	6	0	0.00	60.00	0.0000	0.6000	0.0000	-1.6667
1	4	0	80.00	185.00	0.8000	1.8500		
1	6	0	98.00	208.40	0.9800	2.0840	0.1853	-0.3924
2	3	0	378.00	526.00	3.7800	5.2600	0.0901	-0.1254
2	5	0	282.00	384.00	2.8200	3.8400	0.1242	-0.1692
6	4	0	90.00	206.00	0.9000	2.0600		

Data for Zero sequence (100 MVA base)

Mutual						
Actual					Converted	
P	Q	NO.	RM(%)	XM(%)	RM(pu)	XM(pu)
6	4	0	50.00	95.00	0.5000	0.9500
1	4	0	50.00	95.00	0.5000	0.9500

Existing Mutual Admittance Matrix

Self			Coupled Line			Self		Mutual	
BUS	BUS	NO.	BUS	BUS	NO.	G	B	GM	BM
1	4	0				0.2318	-0.6201		
1	4	0	6	4	0			-0.5895	0.3034
6	4	0				0.2100	-0.5552		

Existing Z matrix for the zero sequence network

Bus	Bus	R	X
1	1	0.36392	1.1134
1	2	0	0
1	3	0	0
1	4	-0.0112	0.1147
1	5	0	0
1	6	0.02526	0.3412
2	2	0	0.032
2	3	0	0.032
2	4	0	0
2	5	0	0.032
2	6	0	0
3	3	3.75	5.292
3	4	0	0
3	5	0	0.032
3	6	0	0
4	4	0.00756	0.2414
4	5	0	0
4	6	-0.01706	0.0555
5	5	2.82	3.872
5	6	0	0
6	6	0.03849	0.0475

APPENDIX B

Matlab code


```
% This code is to perform Fault Analysis of 6-bus system. The sys. data
% was provided by Paul Anderson's book as one of its examples. The
% following functions "Copyright (C) 1998 H. Saadat" were utilized to
% calculate the fault at bus number 4:
% 1)symfaul
% 2)lgfault
% 3)dlgfault
% 4)llfault
clc
```

System Data:

```
% the 6-bus system: line positive sequence parameters
PosSeqData=[ 0, 1, 0.02, 0.24;
            0, 2, 0.03, 0.48;
            1, 4, 0.16, 0.74;
            1, 6, 0.24, 1.036;
            2, 3, 1.446, 2.1;
            2, 5, 0.564, 1.28;
            3, 4, 0, 0.266;
            6, 4, 0.194, 0.814;
            5, 6, 0, 0.6];
% the 6-bus system: line negative sequence parameters
NegSeqData=PosSeqData;
% the 6-bus system: line zero sequence parameters
% without the mutual coupling
ZerSeqData=[ 0, 2, 0, 0.032;
            0, 4, 0, 0.266;
            0, 6, 0, 0.6;
            1, 4, 0.8, 1.85;
            1, 6, 0.98, 2.084;
            2, 3, 3.78, 5.26;
            2, 5, 2.82, 3.84;
            6, 4, 0.90, 2.06];
```

Sequence matrices:

```
% Positive Sequence Matrix:
Z1bus=[ 0.02253+1i*0.21503  -0.00609+1i*0.04974  0.02635+1i*0.16117
0.02254+1i*0.17266  0.02118+1i*0.12825 0.01831+1i*0.16379;
       -0.00609+1i*0.04974   0.04422+1i*0.38094 -0.01594+1i*0.15713 -
0.00786+1i*0.13434 -0.00698+1i*0.22306 0.00023+1i*0.15223;
        0.02635+1i*0.16117  -0.01594+1i*0.15713  0.16244+1i*0.73912
0.14333+1i*0.53368  0.06192+1i*0.27007 0.07295+1i*0.32786;
        0.02254+1i*0.17266  -0.00786+1i*0.13434  0.14333+1i*0.53368
0.13269+1i*0.57694  0.06506+1i*0.27818 0.06881+1i*0.34726;
        0.02118+1i*0.12825  -0.00698+1i*0.22306  0.06192+1i*0.27007
0.06506+1i*0.27818  0.16569+1i*0.80649 0.13256+1i*0.46538;
        0.01831++1i*0.16379  0.00023+1i*0.15223  0.07295+1i*0.32786
0.06881+1i*0.34726  0.13256+1i*0.46538 0.13034+1i*0.61119];
% Negative Sequence Matrix:
Z2bus=Z1bus;
% Zero Sequence Matrix:
Z0bus=[0.36392+i*1.11336  0+1i*0       0+1i*0         -
0.0112+1i*0.11474    0+1i*0          0.02526+1i*0.3412;
    0+1i*0              0+1i*0.032   0+1i*0.032      0+1i*0
0+1i*0.032      0+1i*0 ;
    0+1i*0              0+1i*0.032   3.75+1i*5.292   0+1i*0
0+1i*0.032      0+1i*0            ;
   -0.0112+1i*0.11474   0+1i*0       0+1i*0          0.00756+1i*0.24138
0+1i*0         -0.01706+1i*0.05554;
    0+1i*0              0+1i*0.032   0+1i*0.032      0+1i*0
2.82+1i*3.872   0+1i*0;
    0.02526+1i*0.3412   0+1i*0       0+1i*0         -0.01706+1i*0.05554
0+1i*0          0.03849+1i*0.047472];
```

Fault Analysis Calculations:

```
% pre-fault voltages at all buses:
Vpre=[1.05;1.1;1;1;1;1];
symfaul(PosSeqData,Z1bus,Vpre)
lgfault(ZerSeqData, Z0bus, PosSeqData, Z1bus, NegSeqData, Z2bus, Vpre)
dlgfault(ZerSeqData, Z0bus, PosSeqData, Z1bus, PosSeqData, Z1bus, Vpre)
llfault(PosSeqData, Z1bus, PosSeqData, Z1bus, Vpre)
```

Results:

```
Warning: Invalid escape sequence appears in format string. See help sprintf for valid
escape sequences.
> In symfaul at 34
  In zimp at 57

Balanced three-phase fault at bus No. 4
Total fault current =   1.6892 per unit

Bus Voltages during fault in per unit

     Bus     Voltage       Angle
     No.     Magnitude     degrees
      1       0.7432      -2.2887
      2       0.8629      -4.6653
      3       0.0753      26.7701
```

```
      4       0.0000       0.0000
      5       0.5174       0.1972
      6       0.4027      -2.5905



Line currents for fault at bus No.  4

     From      To     Current     Angle
     Bus       Bus    Magnitude   degrees
      G         1      1.2824    -79.7218
      1         4      0.9816    -80.0882
      1         6      0.3202    -78.8887
      G         2      0.5199    -70.1230
      2         3      0.3136    -62.9284
      2         5      0.2503    -78.0835
      3         4      0.2829    -63.2299
      4         F      1.6892    -77.0478
      5         6      0.1948    -80.1551
      6         4      0.4812    -79.1853

Line-to-ground fault analysis

Single line to-ground fault at bus No. 4

I012 =

   0.1350 - 0.6903i
   0.1350 - 0.6903i
   0.1350 - 0.6903i

Ifabc =

   0.4051 - 2.0709i
   0.0000 + 0.0000i
   0.0000 + 0.0000i


Ifabcm =

    2.1101
    0.0000
    0.0000

Total fault current =    2.1101 per unit

Bus Voltages during the fault in per unit

     Bus    -------Voltage Magnitude-------
     No.    Phase a     Phase b     Phase c
     1      0.7289      1.0422      1.0148
     2      0.9179      1.0360      1.0784
     3      0.2308      0.8999      0.8466
     4      0.0000      0.9413      0.8624
     5      0.5986      0.9232      0.9093
     6      0.4663      0.9312      0.8935
     Bus    ------- Complex Voltage -------
     No.    Zer-Seq     Pos-Seq     Neg-Seq
     1  0.0811 -163.3565  0.9278 -0.4790  0.1225
 -176.3693
     2  0.0000  0.0000  1.0086 -1.3389  0.0947
 -165.5831
     3  0.0000  0.0000  0.6128  2.5134  0.3887
 176.0353
     4  0.1699 -170.7255  0.5840  1.3431  0.4164
 178.1162
     5  0.0000  0.0000  0.7992  0.5267  0.2009
 177.9048
     6  0.0409 -151.8564  0.7510  0.0463  0.2490
 179.8604
Line currents for fault at bus No.  4

    From      To      -----Line Current Magnitude----
     Bus       Bus     Phase a     Phase b     Phase c
```

```
     1        4     0.8871      0.3783      0.3899
     1        6     0.2690      0.1646      0.1657
     2        3     0.2743      0.1365      0.1460
     2        5     0.2318      0.1205      0.1414
     3        4     0.2356      0.1178      0.1178
     4        F     2.1101      0.0000      0.0000


     5        6     0.1622      0.0811      0.0811
     6        4     0.4569      0.1458      0.1458


Zero Sequence currents
Ink0 =

        0                  0                  0             0.0196 - 0.0401i        0
-0.0092 + 0.0156i
        0                  0                  0                  0                  0
0
        0                  0                  0                  0                  0
0
  -0.0196 + 0.0401i        0                  0                  0                  0
-0.0267 + 0.0522i
        0                  0                  0                  0                  0
0
   0.0092 - 0.0156i        0                  0             0.0267 - 0.0522i        0
0

Positive Sequence currents
Ink1 =

        0                  0                  0             0.0683 - 0.4500i        0
0.0299 - 0.1637i
        0                  0             0.0718 - 0.1392i        0             0.0401 -
0.1457i        0
        0           -0.0718 + 0.1392i        0             0.0496 - 0.1069i        0
0
  -0.0683 + 0.4500i        0            -0.0496 + 0.1069i        0                  0
-0.0311 + 0.1980i
        0           -0.0401 + 0.1457i        0                  0                  0
0.0112 - 0.0803i
  -0.0299 + 0.1637i        0                  0             0.0311 - 0.1980i  -0.0112 +
0.0803i        0

Negative Sequence currents
Ink2 =

        0                  0                  0             0.0544 - 0.3855i        0
0.0192 - 0.1179i
        0                  0             0.0496 - 0.1069i        0             0.0112 -
0.0803i        0
        0           -0.0496 + 0.1069i        0             0.0496 - 0.1069i        0
0
  -0.0544 + 0.3855i        0            -0.0496 + 0.1069i        0                  0
-0.0311 + 0.1980i
        0           -0.0112 + 0.0803i        0                  0                  0
0.0112 - 0.0803i
  -0.0192 + 0.1179i        0                  0             0.0311 - 0.1980i  -0.0112 +
0.0803i        0


Double line-to-ground fault analysis

Double line-to-ground fault at bus No. 4
Total fault current =    2.8038 per unit

Bus Voltages during the fault in per unit

     Bus    -------Voltage Magnitude-------
     No.    Phase a     Phase b     Phase c
     1      0.9923      0.7272      0.7507
     2      0.9770      0.9274      0.8887
     3      0.4916      0.2325      0.2744
     4      0.6771      0.0000      0.0000
     5      0.7345      0.5877      0.5709
```

```
     6      0.7201      0.4505      0.4515
     Bus    ------- Complex Voltage -------
     No.    Zer-Seq     Pos-Seq     Neg-Seq
     1  0.1077 13.5156  0.8224 -1.0344  0.0664
 11.6612
     2  0.0000  0.0000  0.9303 -2.7239  0.0513
 22.4473
     3  0.0000  0.0000  0.2816  9.9858  0.2107
  4.0657


     4  0.2257  6.1466  0.2257  6.1466  0.2257
  6.1466
     5  0.0000  0.0000  0.6259  1.1941  0.1089
  5.9352
     6  0.0543 25.0157  0.5360  0.0352  0.1350
  7.8908


zxcLine currents for fault at bus No.  4

     From      To      -----Line Current Magnitude----
     Bus       Bus     Phase a     Phase b     Phase c
      1         4      0.5264      0.9260      0.9424
      1         6      0.2284      0.2870      0.3127
      2         3      0.1950      0.2897      0.3007
      2         5      0.1769      0.2440      0.2474
      3         4      0.1565      0.2506      0.2636
      4         F      0.0000      1.9357      2.1127
      5         6      0.1078      0.1725      0.1815
      6         4      0.1938      0.4715      0.4605
Zero Sequence currents
Ink0 =

        0                  0                  0             -0.0231 + 0.0547i        0
0.0111 - 0.0214i
        0                  0                  0                  0                  0
0
        0                  0                  0                  0                  0
0
   0.0231 - 0.0547i        0                  0                  0                  0
0.0317 - 0.0712i
        0                  0                  0                  0                  0
0
  -0.0111 + 0.0214i        0                  0             -0.0317 + 0.0712i        0
0

Positive Sequence currents
Ink1 =

        0                  0                  0              0.1165 - 0.7827i        0
0.0469 - 0.2655i
        0                  0             0.1149 - 0.2313i        0              0.0500 -
0.2151i       0
        0             -0.1149 + 0.2313i        0              0.0927 - 0.1990i        0
0
  -0.1165 + 0.7827i        0            -0.0927 + 0.1990i        0                  0
-0.0586 + 0.3688i
        0             -0.0500 + 0.2151i        0                  0                  0
0.0212 - 0.1496i
  -0.0469 + 0.2655i        0                  0              0.0586 - 0.3688i  -0.0212 +
0.1496i       0

Negative Sequence currents
Ink2 =

        0                  0                  0             -0.0584 + 0.2028i        0
-0.0193 + 0.0618i
        0                  0            -0.0347 + 0.0536i        0             -0.0121 +
0.0423i       0
        0              0.0347 - 0.0536i        0             -0.0347 + 0.0536i        0
0
   0.0584 - 0.2028i        0             0.0347 - 0.0536i        0                  0
0.0317 - 0.1039i
```

```
        0              0.0121 - 0.0423i        0                  0                  0
-0.0121 + 0.0423i
   0.0193 - 0.0618i        0                  0             -0.0317 + 0.1039i   0.0121 -
0.0423i        0
```

Line-to-line fault analysis

Line-to-line fault at bus No. 4
Total fault current =   1.4629 per unit

Bus Voltages during the fault in per unit

```
     Bus    -------Voltage Magnitude-------
     No.    Phase a     Phase b     Phase c
     1      1.0500      0.8556      0.8250
     2      1.1000      0.9744      0.9099
     3      1.0000      0.4742      0.5325
     4      1.0000      0.5000      0.5000
     5      1.0000      0.6703      0.6726
     6      1.0000      0.6224      0.5965
     Bus    ------- Complex Voltage -------
     No.    Zer-Seq     Pos-Seq     Neg-Seq
     1  0.0000  0.0000  0.9037 -0.8960  0.1471
  5.5145
     2  0.0000  0.0000  0.9914 -1.8439  0.1137
 16.3006
     3  0.0000  0.0000  0.5339  1.8191  0.4667
 -2.0810
     4  0.0000  0.0000  0.5000 -0.0000  0.5000
  0.0000
     5  0.0000  0.0000  0.7587  0.0672  0.2413
 -0.2114
     6  0.0000  0.0000  0.7012 -0.7436  0.2990
  1.7441
```

Line currents for fault at bus No. 4

```
     From      To      -----Line Current Magnitude----
     Bus       Bus     Phase a     Phase b     Phase c
      1         4      0.0660      0.8686      0.8662
      1         6      0.0470      0.2908      0.2895
      2         3      0.0392      0.2817      0.2770
      2         5      0.0715      0.2382      0.2257
      3         4      0.0000      0.2450      0.2450
      4         F      0.0000      1.4629      1.4629
      5         6      0.0000      0.1687      0.1687
      6         4      0.0000      0.4168      0.4168
```

Zero Sequence currents
Ink0 =

```
     0     0     0     0     0     0
     0     0     0     0     0     0
     0     0     0     0     0     0
     0     0     0     0     0     0
     0     0     0     0     0     0
     0     0     0     0     0     0
```

Positive Sequence currents
Ink1 =

```
        0                  0                  0             0.0944 - 0.5250i        0
0.0384 - 0.1866i
        0                  0             0.0859 - 0.1586i        0             0.0455 -
0.1614i        0
        0             -0.0859 + 0.1586i        0             0.0637 - 0.1263i        0
0
  -0.0944 + 0.5250i        0            -0.0637 + 0.1263i        0                  0
-0.0451 + 0.2363i
        0             -0.0455 + 0.1614i        0                  0                  0
0.0167 - 0.0959i
  -0.0384 + 0.1866i        0                  0             0.0451 - 0.2363i  -0.0167 +
0.0959i        0
```

Negative Sequence currents

```
Ink2 =

        0                  0                  0            -0.0805 + 0.4605i        0
-0.0277 + 0.1407i
        0                  0            -0.0637 + 0.1263i        0            -0.0167 +
0.0959i        0
        0            0.0637 - 0.1263i        0            -0.0637 + 0.1263i        0
0
   0.0805 - 0.4605i        0             0.0637 - 0.1263i        0                  0
0.0451 - 0.2363i
        0            0.0167 - 0.0959i        0                  0                  0
-0.0167 + 0.0959i
   0.0277 - 0.1407i        0                  0            -0.0451 + 0.2363i   0.0167 -
0.0959i        0
```

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