Nuclear correlation and finite interaction-range effects in high-energy $(e, e'p)$ nuclear transparency

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Abstract

Nuclear transparency is calculated for high-energy, semi-inclusive $(e, e'p)$ reactions, by accounting for all orders of Glauber multiple-scattering and by using realistic finite-range pN interaction and (dynamically and statistically) correlated nuclear wave functions. The nuclear correlation effect is reduced due to the pN finite-range effect. The net effect is small, and depends sensitively on details of the nuclear correlations in finite nuclei, which are poorly known at present.

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Perturbative quantum chromodynamics (pQCD) predicts the novel nuclear phenomenon of color transparency, in which a hadron produced by a hard scattering process could have an unusually long mean-free-path in nuclei [1]. Such a hadron would be small, color neutral, and suffer little interaction with other nucleons as it goes through the nucleus. The phenomenon has been observed in $(p, 2p)$ [2] and vector meson production [3], while it is not seen in $(e, e'p)$ [4]. pQCD calculations largely remain semi-phenomenological [5] and have not yet yielded precise quantitative predictions.

Low-energy hadrons can have long mean-free-paths in nuclei solely due to nuclear medium effects without color transparency [6]. It is imperative to establish nuclear medium effects reliably in these high-energy reactions, so as to identify the pQCD phenomenon explicitly. The medium effect that has been most noted is nuclear correlation, but calculations vary wildly from only a few percent increase [7] to as much as a 20 to 30 percent increase [8] in proton emission.

The correlation effect results from an interplay between the nuclear reaction mechanism and nuclear structure. As such, both must be formulated and computed with equal rigor and care in order to unambiguously establish the significance of the effect. In this letter we report results of calculations of nuclear transparency in $(e, e'p)$. We include all orders of Glauber multiple-scattering series with realistic finite-range pN interaction together with realistic nuclear correlations, and carry out numerical evaluation using Monte Carlo integration over all nucleon coordinates and spins. No previous work has been carried out with this rigor [9].

The central quantity in this work is the nuclear transparency T, which corresponds to a ratio of the proton emission rate with and without the final-state interaction. T is given as the missing-momentum integration of the response function,

$$S^{\uparrow} = -\frac{1}{\pi} < 0|O^{\dagger} \cdot ImG^{\uparrow} \cdot O|0 > \tag{1}$$

for the ground-state nucleus $|0>$. Here, $ImG^{\uparrow}$ is the imaginary part of the Green's function due to proton emission, and

$$ImG^{\uparrow} = (1 + G^{\dagger}V_{1,A-1}^{\dagger})ImG_0(1 + V_{1,A-1}G), \tag{2}$$

where G_0 and G are the Green's function for the proton in free space and in the nucleus, and $V_{1,A-1}$ is the interaction between the knockout high-momentum proton (labeled as 1) and the rest of the $A-1$ nucleons [10].

We apply the Glauber approximation for Green's functions, which consists of the eikonal approximation for the proton and the fixed-scatterer approximation for the target nucleons, and obtain

$$T = \frac{\int d\mathbf{r}_1 \cdots d\mathbf{r}_A \Pi_{j=2}^{A} e^{-2Im\chi_j(\mathbf{r}_1,\mathbf{r}_j)} |\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A)|^2}{\int d\mathbf{r}_1 \cdots d\mathbf{r}_A |\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A)|^2} \tag{3}$$

with

$$\chi_j(\mathbf{r}_1, \mathbf{r}_j) = -\frac{m}{p_1} \int_{z_1}^{\infty} dz_1{}' V_{pN}(\mathbf{r}_1^{'} - \mathbf{r}_j)_{\mathbf{b}_1^{'}=\mathbf{b}_1}. \tag{4}$$

Equation (3) appeared in our previous work [11] and and also in Ref. [7] as an intermediate step to Glauber multiple-scattering series [12].

T can be cast in the well-known eikonal form expressed in terms of the one-body nuclear density, $\rho(\mathbf{r})$:

$$T = \frac{\int d\mathbf{r}_1 exp(-2Im\chi(\mathbf{r}_1))\rho(\mathbf{r}_1)}{\int d\mathbf{r}_1 \rho(\mathbf{r}_1)}. \tag{5}$$

Here, $2Im\chi(\mathbf{r}_1)$ is expressed in a cluster expansion as

$$\sum_{j=2}^{A} \int d\mathbf{r}_j (2Re\Gamma_j - |\Gamma_j|^2)\rho_2(\mathbf{r}_1, \mathbf{r}_j)/\rho(\mathbf{r}_1) + \cdots, \tag{6}$$

where $\Gamma_j = 1 - exp(+i\chi_j(\mathbf{r}_1, \mathbf{r}_j))$ is the Glauber NN amplitude in the coordinate space.

Nuclear correlation effects appear explicitly in the leading order through the two-body nuclear density ρ_2, and spectator effects appear in the subleading order (not explicitly shown here) [7]. The commonly used eikonal expression is a consequence of invoking the zero-range approximation and setting Γ_j as a constant proportional to the NN cross section, σ: Either

$$2Im\chi(\mathbf{r}_1) \approx \sigma \int_{z_1}^{\infty} dz_1' g(\mathbf{r}_1 - \mathbf{r}'_1)\rho(\mathbf{r}'_1)_{\mathbf{b}'_1=\mathbf{b}_1} \tag{7}$$

or

$$2Im\chi(\mathbf{r}_1) \approx \sigma \int_{z_1}^{\infty} dz'_1 \rho(\mathbf{r'}_1)_{\mathbf{b'}_1=\mathbf{b}_1} \tag{8}$$

with or without the correlation, repectively. Furthermore, many authors use the total cross section, σ_{tot}, as σ. This corresponds to discarding the $|\Gamma_j|^2$ term in Eq. (6) and is not justified for a fully semi-inclusive process [11] [13] since the term is of the leading order through unitarity [12], [11]. Note that the use of σ_{tot} is also inappropriate in exclusive (e,e'p) reactions [14].

V_{pN} in Eq. (4) is either the pp or pn potential, depending on whether the j-th nucleon is the proton or neutron. We determine V_{pN} by applying the Abel transformation to invert the eikonal forms of the NN amplitudes [12]. The inversion is exact in the eikonal formalism for a local, spherical potential; we thus determine V_{pN} better than the frequently used Born approximation. The NN amplitudes are extracted from experimental data in Gaussian forms [15]. Because of the limited availability of pN scattering data, we also restrict the potentials to be spin-independent. The free-space pN interaction would be modified by nuclear medium corrections and by long formation lengths of excited nucleons. These are clearly of much interest, but we proceed by neglecting them in order to determine the correlation effects with minimal complication.

We now describe our findings by taking the case of $p_1 = 4.49$ GeV/c for doubly closed shell nuclei, ^{16}O and ^{40}Ca. This choice is also made to minimize effects of other matters that might complicate the issue, such as open nuclear shells and strong momentum dependence of the pN amplitudes. Note that the p_1 value corresponds to the highest Q^2 in the NE18 $(e, e'p)$ experiment [4].

As Eq. (3) shows, $exp(-2Im\chi(\mathbf{r}, 0))$ contributes to T as a product with the correlated nuclear density. In Fig. 1, we illustrate $exp(-2Im\chi(\mathbf{r}, 0))$ calculated from Eq. (4) using the V_{pp} inverted from the observed pp amplitude. Figure 1 shows that $exp(-2Im\chi(\mathbf{r}, 0))$ deviates from unity even for b and $z > 1$ fm. This feature is missing in the commonly used approximation of the zero-range pN interaction: The corresponding figure in the zero-range approximation would appear as a deep narrow valley following the z-axis starting from $z = 0$.

In order to evaluate T, we require the A-body nuclear density that is accurate in the length scale of several tenths of fm or smaller. Since such densities are not readily available for various nuclei, we construct them by following a simple scheme and compare with a variational Monte Carlo calculation of ^{16}O [16]. We use a Jastrow-form of the wave function: a product of antisymmetrized Hartree-Fock (HF) single-particle wave function (for Skyrme force [17]) $A\Phi$ and of pair wave functions for nuclear matter at various densities [18] f's, which describe the statistical and dynamical correlations, repectively. The A-body density $|\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A)|^2$ is

$$|A\Phi|_s^2 \cdot \prod_{i<j} [(f_c{}^2 + 3f_\sigma{}^2 + 3f_\tau{}^2 + 9f_{\sigma\tau}{}^2 + 6f_t{}^2 + 18f_{t\tau}{}^2) \pm 2(f_c f_\tau + 3f_\sigma f_{\sigma\tau} + 6f_t f_{t\tau} - f_\tau{}^2 - 3f_{\sigma\tau}{}^2 - 6f_{t\tau}{}^2)] \quad (9)$$

for a pn pair $(-)$ and a pp or nn pair $(+)$. Invoking the local density approximation, we evaluate the f's as functions of $|\mathbf{r}_i - \mathbf{r}_j|$, for the (one-body) nuclear density that is generated from $|A\Phi|_s^2$ (spin-summed) at $|\mathbf{r}_i + \mathbf{r}_j|/2$.

Here, we include the dynamical correlations only between nucleon pairs, neglecting correlation operator symmetrization and off-diagonal contributions. The terms depending linearly on the spin-dependent operators are discarded, and those depending quadratically are averaged. This simple scheme should be reasonable for closed-shell nuclei, especially because all noncentral f's are much smaller than the central f_c. The tensor correlations become stronger relative to f_c at short distances ($\lesssim 0.5$ fm), affecting significantly the momentum distribution above the Fermi momentum [20], but this aspect is of little significance in our calculation using Eq. (3).

With these quantities, T is computed by Monte Carlo integration of Eq. (3). The integrations also sample the weight, i. e., the nuclear density. Figure 2 illustrates the two-particle distribution functions, $\rho_{pn}(|\mathbf{r}|)$ and $\rho_{pp}(|\mathbf{r}|)$ for ^{16}O. For comparison, we also show the distribution functions obtained using the $A\Phi$ without the f's. Our ρ_{pn} and ρ_{pp} have the same shapes as those by the variational Monte Carlo calculation [16] with the peak being at the same location. But our peak heights are lower than theirs by about 40 %, and our ρ_{pn}

and ρ_{pp} calculated from $A\Phi$ alone are also lower than their mean-field results by a similar amount. Our one-body density changes little by the application of f's and remains close to observation, while the one-body density by the variational calculation changes substantially.

Figures 1 and 2 illustrate our main finding in this letter that the finite interaction-range effect reduces the nuclear correlation effect [19]. We elaborate on this in the followings.

The values of T are tabulated in Table I, computed by the use of Eqs. (3) and (5) under various assumptions. T using Eqs. (7) and (8) is referred to as *eikonal* and is calculated with the one-body density obtained from Eq. (9). We show a)the most naive but popular calculation using σ_{tot} and b) also with $g(r) = 1 - exp(-(r/a)^2)$ ($a = 0.7$ fm). T is increased from a) to b) by about 30 % and becomes accidentally rather close to that of our full result in 6), but this hides the essential physics that we are addressing.

In 1) we show the eikonal result by the use of σ_{inel}. This corresponds to the leading-order [Eq. (6)] calculation of our full expression [Eq. (3)] under the zero-range approximation without correlation, as it has been noted previously [11] [7] [13]. 1) is the one that should be compared with Monte Carlo results of the full expression, 2) – 6) labeled as GMS, *Glauber multiple-scattering*. 2) – 5) examine separately effects of the dynamical and statistical correlations. Once the finite-range interaction effect is included, we find a little variation among T's of 2) – 6). 6) is our full calculation. By comparing 1) and 6), we see that the net effect due to both nuclear correlation and finite-range interaction is of several %, above the eikonal T with σ_{inel}. This establishes our main finding. Note that the net effect becomes less in the heavier nucleus.

In the rest of this Letter, we demonstrate that our main finding is affected little by the choice of the dynamical NN correlations. As noted above, the peak heights of our ρ_{pn} and ρ_{pp} differ from those by the variational calculation. While ours remain close to the observation within about 1 %, the RMS radius of the variational ^{16}O density is smaller than the observed by about 8 %. When we artificially enhanced our f's so as to obtain the variational ρ_{pn} and ρ_{pp}, we observed our RMS radius decreased as much as the variational density did (though ours had a shallower central depression.) T computed with the enhanced f's is 0.548 $\pm$

0.003, which is *smaller* than our full calculation of 6), 0.577 ± 0.002. That is, the enhanced f's causes nuclei to be *less* transparent.

The reason for the effect is as follows: At a low nuclear density, f_c around 1 - 2 fm becomes larger than the asymptotic value of unity, caused by the attractive, intermediate-range NN interaction [18]. The large f_c lifts the peaks in the two-particle distributions above the mean field results and increases the one-body density. Consequently, the knockout proton suffers more final-state interaction. The variational one-body density differs from observation, and our wave function is not constructed for the energy minimization. We conclude that the two-particle distributions are not reliably known, causing to be T uncertain, perhaps at about the 5 % level.

In order to demonstarte this point, we also use a schematic f_c, $1 - exp(-(r/a)^2)$ ($a =$ 0.7 fm) by setting all other f's zero. For this, ρ_{pn} and ρ_{pp} are found to be always below the HF distributions, the RMS of the density increases from the HF value of 2.65 to 2.71 fm, and T increases to 0.605 ± 0.003. What the correlation does in this case is merely to enlarge the nuclei and to make them more transparent. Note that the correlation effect usually discussed in the literature includes only this short-range repulsive aspect.

In this work, we have examined the reaction that is fully semi-inclusive. That is, the E_m integration is over the full range, and this permits the use of closure to simplify our calculation. A Monte Carlo simulation is underway to examine how realistic this procedure is by comparing with the NE18 experiment setup [21]. Our calculations reported here are also for simplified kinematics without missing momentum ($\mathbf{p}_m$) cut. Our formalism allows $\mathbf{p}_m$ to be finite, though Eq. (3) becomes more complicated, depending on one-particle off-diagonal nuclear density and the real part of V_{pN}. Furthermore, our formalism is also applicable to *inclusive* (e, e'), involving the correlations differently from the *semi-inclusive* $(e, e'p)$ case considered here. We will describe a detailed account of the present work and these considerations in forthcoming publications.

In conclusion, we find that 1) the finite-range pN interaction reduces the effect of nuclear correlation, 2) the net effect is small and becomes smaller in heavier nuclei, and 3) the precise value of the transparency above the eikonal value depends sensitively on details of the dynamical correlations in finite nuclei, which are presently known rather poorly.

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TABLES

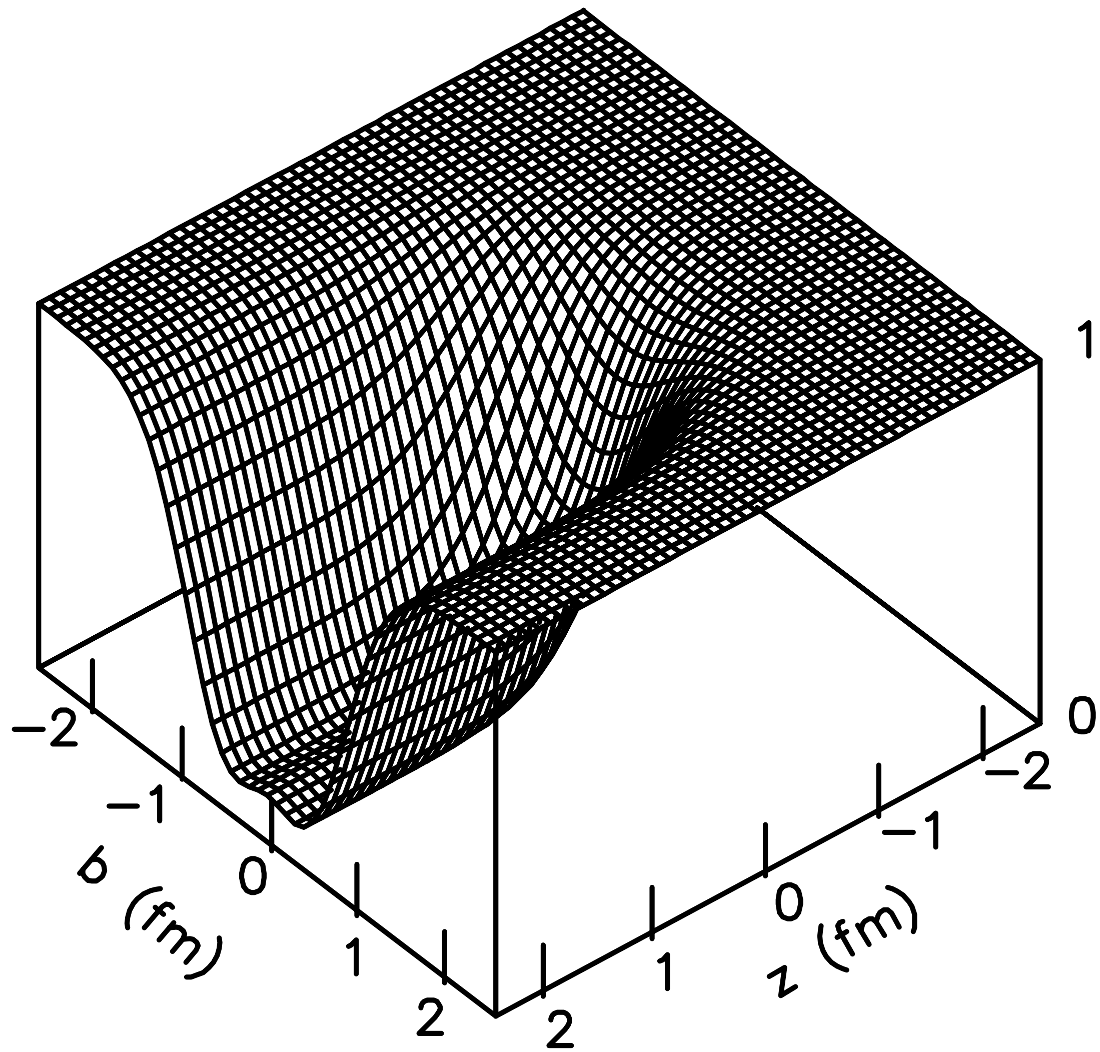
TABLE I. Nuclear transparency of closed-shell nuclei at the knockout proton momentum 4.49 GeV/c in various approximations and models: With or without pN finite-range interaction (FRI), dynamical correlation (DC), and statistical correlation (SC) in the eikonal and Glauber multiple-scattering (GMS) formalism. The numbers in parentheses denote equations used. g denotes the use of the simple form of the pair-distribution function (shown in text) and POD of a product of one-body densities generated by integarting the A-body density indicated. 5) excludes DC among spectators (not involving the knockout proton.) Single-digit values in parentheses are statistical uncertainties (in the last digits) in Monte Carlo integrations.

	FRI	DC	SC	Comments	^{16}O	^{40}Ca
a)				Eikonal$[((7);\sigma_{tot}]$	0.444	0.335
b)				Eikonal$[(8);\sigma_{tot},g]$	0.529	0.405
1)	Zero	No	No	Eikonal$[(7);\sigma_{inel}]$	0.560	0.446
2)	Finite	No	No	GMS$[\text{POD};(\lvert A\Psi\rvert^2)]$	0.570(2)	0.439(2)
3)	Finite	Yes	No	GMS$[\text{POD};(9)]$	0.568(2)	0.442(3)
4)	Finite	No	Yes	GMS$[\lvert A\Psi\rvert^2]$	0.579(3)	0.447(3)
5)	Finite	Yes	Yes	GMS$[(9);\text{Nospect}]$	0.582(2)	0.450(3)
6)	Finite	Yes	Yes	GMS$[(9);\text{Full}]$	0.577(2)	0.447(3)

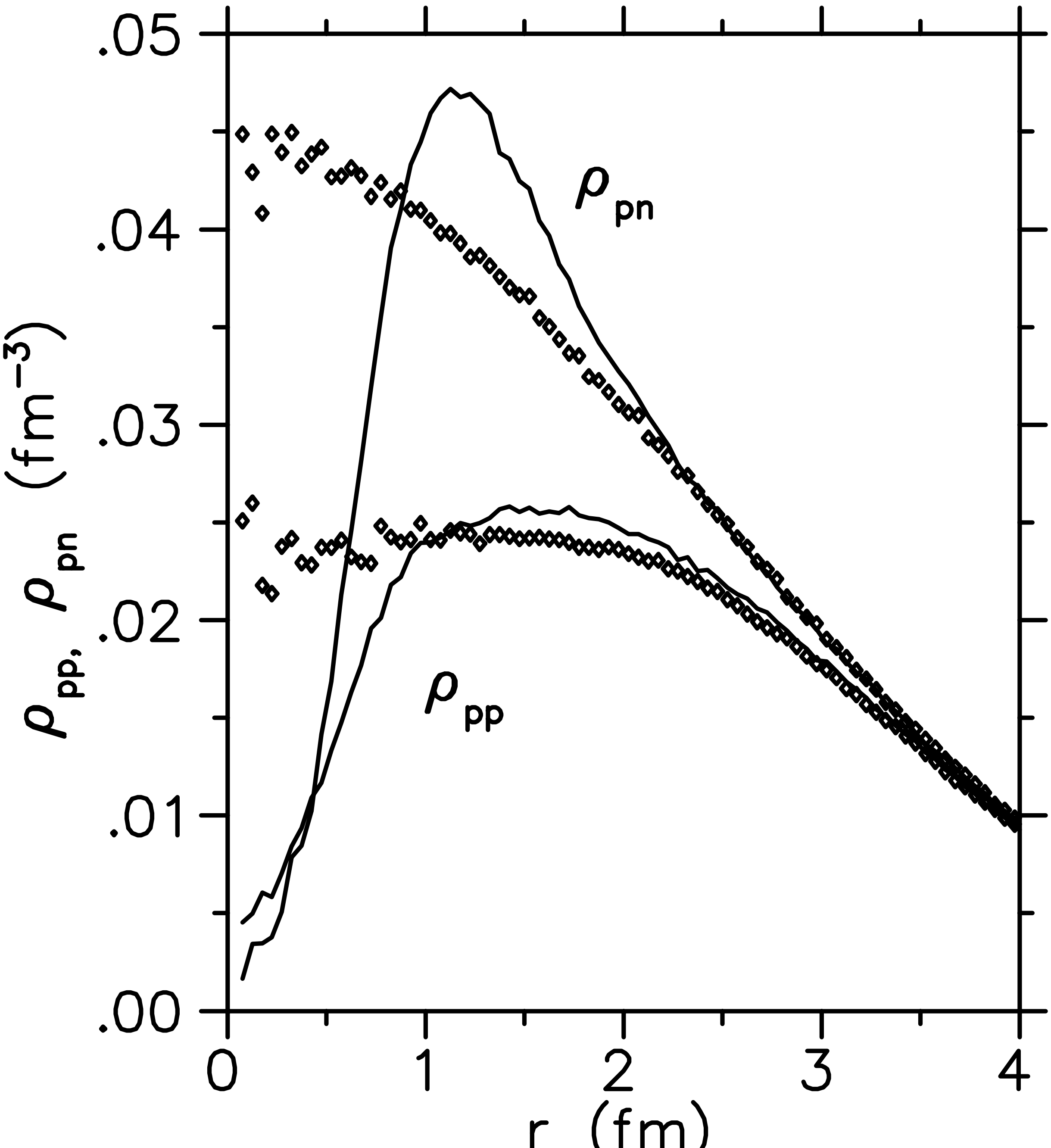
FIGURES

FIG. 1. $exp(-2Im\chi(\mathbf{r},0))$ using the spin-independent, local V_{pp} extracted from experimentally determined pp scattering amplitude. The quantity illustrates attenuation of the knockout proton as it impinge on a proton located at the origin. The proton comes from the negative z, parallel to the z-axis.

FIG. 2. The two-particle distributions of ^{16}O, ρ_{pn} and ρ_{pp}, calculated using the wave function of Eq. (9) with and without pair correlation functions, shown by full curves and open circles, respectively.



b (fm)
z (fm)
−2
−1
0
1
2
−2
−1
0
1
2
0
1


.05
.04
.03
.02
.01
.00
0
1
2
3
4
r (fm)
ρ_{pp}, ρ_{pn} (fm^{-3})
ρ_{pn}
ρ_{pp}