Modified 4D Laplacian for Smooth Pressure Reconstruction Based on Time-Resolved Velocimetry (2): Experiments

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Abstract

In this work, a velocimetry-based pressure reconstruction technique that takes advantage of the smoothing nature of the inverse Laplacian to remove the high-frequency noise from the reconstructed instantaneous pressure fields is studied. This pressure reconstruction technique (implemented in DaVis 10 by LaVision) modifies the Laplacian in the pressure Poisson equation by adding a temporal diffusion term multiplied by a weighting factor. In order to reduce the computational cost for long-time-series data, the temporal dimension is split into several space-time blocks. Time-resolved tomographic PIV measurements of an impinging synthetic jet are used to reconstruct the pressure fields. While allowing diffusion in time mitigates the high-frequency temporal noise from the pressure time series, it introduces a positive definite temporal drift. The drift is inherent to the modified Laplacian and is affected by the weighting factor of the temporal diffusion term and the number of the space time blocks. The temporal resolution of the velocity data and the flow oscillation also affect the drift. This study shows that, with a proper selection of different parameters, it is possible to remove the non-physical high-frequency noise from the pressure fields and limit the temporal drift.

1 Introduction

Instantaneous pressure fields reconstructed from time-resolved PIV data contain high-frequency temporal noise due to error propagation from the velocity field measurements (Charonko et al., 2010; De Kat and Van Oudheusden, 2012). This noise in the pressure fields manifests itself as overestimation of the power spectral density at higher frequencies for both Poisson equation based (De Kat and Van Oudheusden, 2012; Van Oudheusden, 2013; Ghaemi et al., 2012) and direct integration based pressure reconstruction (Liu and Moreto, 2020). The techniques used to remove this high frequency temporal noise from the pressure field either 1) suppress the noise at the source (experimental measurements, calculation of the pressure gradient, calculation of the source term of the pressure Poisson equation etc.) or 2) use a smoothing technique to filter out the noise during the pressure field reconstruction.

One example of the latter strategy, filtering the pressure by reconstruction and implemented in DaVis 10.2 by LaVision, uses a modified form of the pressure Poisson equation (1) where the Laplace operator includes an additional diffusion term in the temporal dimension,

\[ \tilde{\nabla}^2 p := \nabla^2 p + \xi \frac{\partial^2 p}{\partial t^2} \bigg|_c = f(u(x,t)), \]

where \( \tilde{\nabla}^2 \) is a modified Laplace operator and \( \xi \) is the weighting factor between the temporal and spatial derivative of the pressure at an instant. The estimation of the temporal derivative of pressure in this formulation requires knowledge of the convection velocity of the flow structures. The convection velocity of...
the flow structures provides the grid location between time steps from which the temporal derivative of the pressure is calculated. The solution of equation (1) for a longer duration of time-resolved data is computationally expensive due to the additional dimension in time and, in turn, a large number of calculations and large memory requirement. Therefore, the modified pressure Poisson solver in DaVis 10.2 divides the computational domain into multiple space-time blocks to overcome this issue. The solver allows the user to control the weight of the temporal diffusion term ($\xi^2$) and the number of space-time blocks ($m$) to balance the smoothness, accuracy, and computational cost.

This modified formulation of the pressure Poisson equation takes advantage of the low-pass filter behavior of inverting the Laplace operator (Charonko et al., 2010; Faiella et al., 2021) in the temporal dimension to filter out the high-frequency noise. The use of the smoothing property of the Laplacian can also be found in the field of image processing (Zhang et al., 2023) for seamless stitching of images (Levin et al., 2004), smooth inpainting (Bertalmio et al., 2001) and temporal smoothing of videos (Bonneel et al., 2015).

While this modified formulation of the pressure Poisson equation can eliminate the high-frequency temporal noise in the reconstructed pressure field for time-resolved data, Sakib et al. (2021) demonstrated that it results in a nonphysical temporal drift for oscillating internal flows. Additionally, this study showed that the slope of the temporal drift scales with the temporal resolution of the oscillating flow.

A recent analytical study by Zhang et al. (2023) models the error bounds and smoothing effects of the modified formulation of the pressure Poisson equation. This work explains that the temporal drift is inherent to the modified pressure Poisson equation and that it is related to the frequency of the oscillation of the flow, the weighting factor ($\xi$), and the number of the space-time blocks. In the current work, time-resolved tomo-PIV data obtained in an impinging synthetic jet experiment are used to experimentally investigate the effect of different parameters on the computed pressure field using the modified pressure Poisson solver.

2 Experiments

The experimental setup consists of a hexagonal water tank fitted with a piston-cylinder device at the bottom. The movement of the piston is controlled by an electromagnetic shaker and pushes water through an orifice to generate a vertical synthetic jet that impinges on a horizontal plate located 15 diameters above the orifice. The impingement plate is fitted with two pressure sensors, S1 and S2, that record the pressure at the impingement location and at the far field. The synchronized S1 and S2 pressure sensor data are used as a validation measurement and a Dirichlet boundary condition, respectively. Figure 1 shows the schematic of the experimental setup.

![Figure 1: Schematic of the synthetic jet impingement experiment](image)

Two synthetic jet flow conditions with similar impingement pressures but distinct frequencies (10 Hz and 5 Hz) are studied. The tomo-PIV measurements are acquired with four high-speed cameras at 1,000
frames per second for both cases. The same acquisition frequency for both synthetic jets frequencies results in higher non-dimensional temporal resolution for the 5 Hz jet compared to the 10 Hz jet. Due to the memory and processing constraints, the experiment duration is limited to six cycles of the jets.

An iterative volume reconstruction technique known as the fast Multiplicative Algebraic Reconstruction Technique (fast MART), as implemented in DaVis 10.2 is used to reconstruct the 3D intensity distribution of the seeding particles. Instantaneous velocity fields are obtained using a multi-pass, 3D cross-correlation algorithm with a final interrogation volume of $32 \times 32 \times 32$ voxels with 75% correlation volume overlap which results in a vector spacing of 0.4 mm. Figure 2a shows the velocity field through the mid xy-plane of the measurement domain and figure 2b shows iso-surfaces of the v velocity in the 3D domain respectively.

The velocity field (figure 2a) indicates a nearly quiescent condition in the far field, so a zero Dirichlet boundary condition for the pressure field can be prescribed at this location. In this study, four Dirichlet boundary conditions are prescribed as zero pressure at the four corners of the domain and at the rest of the boundary, Neumann conditions (pressure gradient normal to the boundary) are calculated directly using the Navier-Stokes equation. The pressure fields are reconstructed with the modified Laplacian solver for various combinations of the weighting factor ($\xi$) and the number of space-time blocks ($m$).

![Figure 2: An instantaneous velocity field obtained by tomo-PIV for the 10 Hz jet. a) Slice of the vector field through the mid xy–plane of the measurement volume. The blue and red colors are the out-of-plane clockwise and counter-clockwise vorticity, respectively. b) 3D iso-surfaces of the v velocity. c) Iso surfaces of the pressure field reconstructed with the 4D solver. The vector field is shown at mid xy-plane](image)

3 Results and Discussion

The pressure fields are reconstructed with the modified pressure Poisson solver for both synthetic jet cases for various combinations of the weighting factor $\xi$ and number of space-time blocks $m$. Figure 2c shows the 3D iso-surfaces of the pressure field with the vector field at the mid-xy-plane for the 10 Hz jet. This pressure field is reconstructed using $\xi = 1$ and $m = 600$. The pressure field is consistent with the vector field and preserves the flow structures well.

Figure 3 shows the colormap of the pressure field for the 10 Hz jet at the same phase location for cycles 1, 3 and 6. The saturation of the color with time indicate a temporal drift. The 5 Hz jet shows a similar temporal drift in the pressure field. The reason for this positive definite temporal drift in the pressure field in these tests is attributed to the oscillating nature of the flow and the fundamental diffusive property of the modified Laplacian based 4D solver, coupled with the improper selection of the weighting factor and space time block size.

It is noted that other users of this solver (Michaelis and Wieneke 2019; Saiz et al. 2022) did not report any such drift. Saiz et al. (2022) reconstructed the pressure field in small regions located outside the shear layer of a flapping plate which is located behind a cylinder. The relatively low frequency of oscillation and short recording duration may be the reason for no observable temporal drift in this study. Michaelis and Wieneke (2019) reconstructed the pressure field for a steady jet and compared the results obtained using different volumetric velocimetry techniques. This flow is not pulsatile in nature and the recording period is also short. However, the drifting was also observed by the developer of the solver (Jeon, 4/21/2023).

Figure 4 shows the impingement pressure for different values of the weighting factor for the 10 Hz jet and 5 Hz jet with the number of space-time blocks $m = 600$. While the high-frequency temporal fluctuation
decreases with a higher value of the weighting factor, in this particular setup, the temporal drift increases with the increase of the weighting factor. A comparison between the 10 Hz jet (4a) and the 5 Hz jet (4b) shows that for the same value of the weighting factor the apparent temporal drift is lower for the 5 Hz jet which has a higher non-dimensional temporal resolution. While the relationship between the temporal resolution and the drift is not explained by Sakib et al. (2021), Zhang et al. (2023), shows higher frequency of oscillation results in higher error, which in turn may result in high temporal drift.

Figure 3: Pressure field reconstructed with the 4D solver for the 10 Hz jet at $t/T = 0.3$ for (a) cycle 1, (b) cycle 3 and (c) cycle 6. The saturation of the color in the pressure field demonstrate the temporal drift.

For a particular jet, the higher value of $\xi$ allows higher diffusion of the error in time which results in smoother pressure in time. However, the error analysis of the modified Poisson equation performed by Zhang et al. (2023) show that the error in the pressure field may increase with increasing $\xi$ and this leads to higher temporal drift. Additionally, the smoothing effect of higher value of the weighting factor will also smear the pressure field itself. Therefore, an appropriate choice of the weighting factor is a result of trade-off among these effects.

Figure 4: Impingement pressure calculated with different values of the weighting factor $\xi$ for the a) 10 Hz jet and b) 5 Hz jet.

The modified Laplacian based pressure solver implemented in DaVis 10 splits the temporal dimension into multiple space-time blocks to reduce the computational cost. The effect of splitting the time dimension is studied by reconstructing the pressure field for different number of space-time blocks for a fixed value of the weighting factor. Figure 5 shows the impingement pressure for the 10 Hz jet for different number of space-time blocks for $\xi = 1$ (5a) and $\xi = 0.5$ (5b). It can be observed that, for a fixed value of the weighting factor, the temporal drift decreases with the decrease in the number of space-time blocks. A small number
of space-time blocks means each block is longer (measured by the length in time), therefore, more time is available to diffuse the error accumulated from the previous space-time block, which results in a smaller drift.

Figure 5: Impingement pressure for the 10 Hz jet for different number of space-time blocks calculated with the weighting factors a) $\xi = 1.0$ and b) $\xi = 0.5$.

The above discussion on the effect of the weighting factor and the number of space-time blocks on the pressure reconstruction show that, a relatively smaller value of the weighting factor with longer space-time block size can reduce the non-physical high-frequency temporal fluctuation from the pressure fields and keep the temporal drift minimum. One way to ensure an appropriate choice of these parameters is to compare the power spectral density of the reconstructed pressure time series with that of a physical measurement. Van Gent et al. (2018) used a similar strategy to optimize the number of frames used in the material acceleration calculation during pressure PIV reconstruction.

Figure 6 shows the power spectral density of the impingement pressure calculated with the 3D Poisson solver and the modified Laplacian solver and compares it with the power spectral density of the pressure sensor measurement. The impingement pressure calculated with the 3D solver (blue line) shows an over-estimation of the power spectral density at higher frequencies compared to the physical measurement (red line). The power spectral density of the impingement pressure obtained with the modified Laplacian solver ($\xi = 1, m = 12$) follows the physical measurement more closely.

Figure 6: Comparison of Power spectral density of the impingement pressure for the 10 Hz jet calculated with different pressure solvers with the pressure sensor data.
The weighting factor effect and the space-time block size effect observed in this study is obtained for an oscillatory flow with mostly Neumann type boundary conditions. The flow type and the boundary conditions effect (which plays a significant role in pressure-PIV reconstruction) coupled with the weighting factor and space-time block size effect may affect the propagation and the accumulation of error.

4 Conclusion

This work presents a parametric study on a PIV pressure reconstruction technique that is based on adding a temporal diffusion term in the pressure Poisson equation to remove the high-frequency error from the pressure fields. Time-resolved tomo-PIV data obtained in an impinging synthetic jet experiment are used to reconstruct the instantaneous pressure fields. While this technique can remove the high-frequency temporal noise from the pressure field, it may result in a positive definite drift in time. The drift is affected by the weighting factor of the diffusion term and the size of the space-time blocks. A large value of the weighting factor results in smoother pressure fields in time, however, it increases the temporal drift. A larger space-time block size allows more diffusion of the error between time steps and lowers the temporal drift. It is possible to select these parameters in such a manner that eliminates the high-frequency error from the pressure field while keeping the drift minimum.

References


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